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THE RIEMANN METHOD IN R^n FOR A SYSTEM WITH MULTIPLE CHARACTERISTICS

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In areas which are typical for the Goursat and Cauchy problems we consider the system of equations with multiple characteristics

$$u_{lx_j} = \sum_{i=1}^{m} a_{li}(x_1,\dots,x_n)u_i + f_l(x_1,\dots,x_n), \quad 1 \leqslant l \leqslant m = \sum_{i=1}^{n} k_i, \tag{1}$$

if $1 \leqslant l \leqslant k_1$, then $j=1$, if $k_1+1 \leqslant l \leqslant k_1+k_2$, then $j=2$, if $k_1+k_2+1 \leqslant l \leqslant k_1+k_2+k_3$, then $j=3$, ... , if $\sum\limits_{i=1}^{n-1} k_i + 1 \leqslant l \leqslant \sum\limits_{i=1}^{n} k_i$, then $j=n$. Hereinafter we assume that all a_{li}, f_l are continuous in the closure of the area under consideration. We call a solution of (1) regular in an area D if it is continuous in D, as well as all the derivatives which enter into the system: $u_l \in C(D)$, $l=\overline{1,m}$, $u_{lx_j} \in C(D)$, $\sum\limits_{i=1}^{j-1} k_i + 1 \leqslant l \leqslant \sum\limits_{i=1}^{j} k_i$.

Similar systems with non-multiple characteristics are studied in [1], [2]. In [1] by the method of successive approximations one obtains formulas for a solution of the Goursat problem for the system

$$u_{lx_l} = \sum_{i=1}^{n} a_{li}(x_1,\dots,x_n)u_i + f_l(x_1,\dots,x_n), \quad l=1,\dots,n. \tag{2}$$

In [2] one investigates the Cauchy and Goursat problems for system (2) with $n=2$; in particular, one proposes the formulas of the integral representation of solutions which enable one to establish their structural properties.

In this paper, we propose another approach to constructing solutions of the Cauchy and Goursat problems. Namely, we propose a version of the Riemann method which develops ideas of [3], [4], where one studies systems of equations with two independent variables. However, as distinct from [3], we define the Riemann matrix as a solution of a system of integral equations. In [5]–[13], the Riemann functions are introduced in a similar way for one equation.

1. Let $G = \{x_i^0 < x_i < x_i^1,\ i=\overline{1,n}\}$. Denote by X_j the faces of G with $x_j = x_j^0$.

The Goursat problem. Find a regular in an area G solution of system (1) which satisfies the conditions

$$u_l|_{X_j} = \varphi_l(x_1,\dots,x_{j-1},x_{j+1},\dots,x_n), \quad l=\overline{1,m}, \tag{3}$$

$\varphi_l \in C(\overline{X}_j)$, l and j are connected by formula (1).