ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 2, pp. 39–45. © Allerton Press, Inc., 2011.
Original Russian Text © I.V. Konnov and I.A. Pastukhov, 2011, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2011, No. 2, pp. 46–53.

A Method for Solving a General Multi-Valued Complementarity Problem

I. V. Konnov* **and I. A. Pastukhov†**

Kazan State University, ul. Kremlyovskaya 18, Kazan, 420008 Russia

Received June 8, 2009

Abstract—We propose an extended version of Chandrasekaran's method for general complementarity problems with multi-valued weakly off-diagonally antitone cost mappings. It allows one either to construct a sequence converging to a solution or to recognize that the problem has no solutions. We also propose versions of Jacobi's methods for multi-valued inclusions subject to one- and two-side constraints.

DOI: 10.3103/S1066369X11020058

Keywords and phrases: *complementarity problem, multi-valued mapping, off-diagonal antitonicity, coordinate descent method, multi-valued inclusions.*

1. INTRODUCTION

The complementarity problem is now one of the basic problems in nonlinear analysis together with optimization, fixed point, and variational inequality problems. A great number of works is devoted to its theory, solution methods, and applications; see, e.g., [1–6] and references therein. We recall that the classical *nonlinear complementarity problem* consists in finding a point $x^* \in R^n$ such that

$$x^* \geq \mathbf{0}, F(x^*) \geq \mathbf{0}, \ \ \langle x^*, F(x^*)\rangle = 0; \tag{1}$$

where $F : R^n \to R^n$ is a given single-valued mapping (here and below the vector inequalities are regarded as coordinate-wise, $\mathbf{0}$ denotes the zero vector). Most results in this field especially on numerical solution methods are related just to the problems in the form (1); see, e.g., [2, 5]. The most investigated one is the case of the linear complementarity problem where the mapping F is affine.

At the same time, many applications, in particular in Economics and Mechanics, force one to solve such problems involving multi-valued mappings; see, e.g., [7–10] and references therein. We recall that the general *multi-valued complementarity problem* consists in finding a point $x^* \in R^n$ such that

$$x^* \geq \mathbf{0}; \ \exists g^* \in G(x^*), \ g^* \geq \mathbf{0}; \ \langle x^*, g^*\rangle = 0; \tag{2}$$

where $G : R^n \to \Pi(R^n)$ is a given multi-valued mapping (here and below $\Pi(S)$ denotes the family of all subsets of a set S). It is well known (see, e.g., [11], lemma 1) that problem (2) coincides with the multi-valued variational inequality where the feasible set is of the form

$$R^n_+ = \{x \in R^n \mid x \geq \mathbf{0}\},$$

i.e., it is the non-negative orthant in R^n. On account of the special feasible set one can utilize weaker order monotonicity properties instead of the usual norm monotonicity ones.

Recently, in [10–14], concepts of the off-diagonal antitonicity were extended to multi-valued mappings, results on existence of solutions for the corresponding multi-valued complementarity problems were obtained, and extensions of Jacobi and Gauss–Seidel type methods were constructed for these

† Deceased.

* E-mail: konn-igor@yandex.ru.