

ON ESTIMATION OF RESOLVENT  
OF SECOND ORDER DIFFERENTIAL OPERATOR  
WITH NON-REGULAR BOUNDARY CONDITIONS

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**1.** Let  $ly = -y'' + a_1(x)y' + a_2(x)y$  be a linear differential expression,  $x \in [0, 1]$ ,  $a_1(x)$  and  $a_2(x)$  continuous on  $[0, 1]$  functions, while  $L_1y$  and  $L_2y$  the corresponding boundary forms. In addition, we shall assume that they contain some integrals of the function  $y = y(x)$ .

Ordinary differential equations with nonlocal boundary conditions  $L_iy = 0$  ( $i = 1, 2$ ) were studied by many authors (see, e. g., [1]–[6]). In the referred papers, various spectral properties of the corresponding operator were considered (spectrality, eigenfunctions, conjugate problem, preferably in the space  $L_2(0, 1)$ ); moreover, the boundary forms contain the values of the function  $y(x)$  or its derivatives on the ends of segment. This allows to construct a conjugate operator (see [2]) or assume the regularity (see [7], p. 67) of the boundary value conditions (see [1], [4]). Our interest is in the question on the regular set of the operator  $L$ , which corresponds to the differential expression  $ly$ , and behavior of its resolvent  $(L + \lambda I)^{-1}$  for large values of  $\lambda$  in dependence on the boundary conditions in the space  $L_p(0, 1) = L_p$ . If the boundary conditions for  $L$  are regular (see [7], p. 67), then the regular set  $\lambda(L)$  of the operator  $L$  contains the domain

$$\Omega_\varepsilon = \{\lambda : |\arg \lambda| < \pi - \varepsilon, |\lambda| \geq R_\varepsilon\}$$

for any  $\varepsilon > 0$  and  $\|(L + \lambda I)^{-1}\| \leq c|\lambda|^{-1}$ .

In [8], nonlocal boundary conditions were considered, which contained values of the desired function at the ends of the segment and which are not regular. In this situation, it was shown that  $\Omega_\varepsilon \subset \rho(L)$  and  $\|(L + \lambda I)^{-1}\| \leq c|\lambda|^{-1/2}$ . Finally, in [5], [6] a necessary and sufficient condition for the spectrum of operator  $L$  be discrete was established in the case of integral conditions

$$L_iy = \int_0^1 \varphi_i(x)y(x)dx, \quad i = 1, 2, \quad (1)$$

and for regular  $\lambda$  for  $p = 2$  the estimate was obtained

$$\|(L + \lambda I)^{-1}\| \leq c|\lambda|^{-3/4}.$$

In the present article the differential operator  $L$  is considered with the boundary conditions (1). For this operator we establish more general sufficient conditions of the existence of resolvent, as well as give estimation of its behavior in the space  $L_p$  for any  $p \geq 1$ .

**2.** For construction of the resolvent of the operator  $L$  we consider the differential equation

$$Ly + \lambda y = f(x) \quad (2)$$

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