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A Semiexplicit Method for Numerical Solution of Functional Differential Algebraic Equations

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Abstract—We study systems with delay effect that contain additional algebraic relations. We propose semiexplicit numerical methods of the Rosenbrock type. We prove the solvability of equations of a numerical model and estimate the order of the global error. The chosen parameters provide the third order of the error.

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1. INTRODUCTION

Differential equations with additional algebraic constraints are closely connected with stiff singular ordinary differential equations (ODE) and are widely applied in the mathematical modeling. Numerical solution methods for such differential-algebraic equations (DAE) were studied in monograph [1]. However, in many models of the ambient reality future states of a system depend not only on the present one, as in ODE, but also on the past states. This effect is mathematically described by equations with aftereffects of various kinds; they are also called functional differential equations (FDE). Qualitative aspects of the theory of FDE were studied by many authors, see [2]–[4] for several numerical solution methods. The combination of these two effects, namely, the presence of additional algebraic connections and an aftereffect, gives a new object, namely, systems of functional differential-algebraic equations (FDAE). See [5] for methods of the Runge–Kutta type for the numerical solution of FDAE, see [6] for multistep numerical methods. The mentioned papers use one technique proposed earlier in [4] for the solution of equations with delay. This technique is based on the following main ideas: The separation of the finite- and infinite-dimensional components in the structure of the phase state of the object; the development (by the finite-dimensional component) for FDE of full analogs of numerical methods applied to ODE; the interpolation of the discrete prehistory of a model with given properties in order to take into account the infinite-dimensional component; the use of the technique of the i-smooth analysis (see [4]) for obtaining analogs of the Taylor decomposition of functionals that are necessary for the analysis of the local error in the development of a method.

In this paper on the base of the mentioned analogs as applied to FDAE we construct and study methods that are known as methods of the Rosenbrock type for ODE (see [1, 3]). The Rosenbrock-type methods belong to a wide class of methods, where one does not have to solve nonlinear systems, replacing them with a sequence of linear ones; therefore these techniques are often called linearly implicit Runge–Kutta methods.

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