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The Boundary-Value Problem for the Lavrent'ev–Bitsadze Equation with Unknown Right-Hand Side

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Abstract—We study the inverse problem for the Lavrent'ev–Bitsadze equation in a rectangular domain. We construct its solution as a series of eigenfunctions for the corresponding problem on eigenvalues and establish a criterion for its uniqueness. We also prove the stability of the obtained solution.

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1. Introduction. Consider the following mixed-type Lavrent'ev–Bitsadze equation:

$$Lu \equiv u_{xx} + (\operatorname{sgn} y)u_{yy} = f(x) \tag{1}$$

in the rectangular domain $D = \{(x,y) \mid 0 < x < 1,\ -\alpha < y < \beta\}$, where α and β are given positive numbers, as well as the boundary value problem analogous to that stated and studied in [1].

The inverse problem. *Find in the domain D functions $u(x,y)$ and $f(x)$ such that*

$$u \in C^1(\overline{D}) \cap C^2(D_+ \cup D_-); \tag{2}$$

$$f(x) \in C(0,1) \cap L[0,1]; \tag{3}$$

$$Lu(x,y) \equiv f(x),\ \ (x,y) \in D_+ \cup D_-; \tag{4}$$

$$u(0,y) = u(1,y) = 0,\ \ -\alpha \le y \le \beta; \tag{5}$$

$$u(x,\beta) = \varphi(x),\ \ u(x,-\alpha) = \psi(x),\ \ 0 \le x \le 1; \tag{6}$$

$$u_y(x,-\alpha) = g(x),\ \ 0 \le x \le 1, \tag{7}$$

where φ, ψ, and g are given sufficiently smooth functions, $\psi(0) = \psi(1) = 0$, $\varphi(0) = \varphi(1) = 0$, $D_+ = D \cap \{y > 0\}$, and $D_- = D \cap \{y < 0\}$.

Inverse problems for hyperbolic, parabolic, and elliptic differential equations were studied by many authors [2–6]. In papers [1] and [7–9] one has commenced studying inverse boundary-value problems for mixed-type parabolic-hyperbolic and elliptic-hyperbolic equations by reducing the inverse problem to the direct one. In this paper we propose a variant of the inverse problem connected with the search of the right-hand side for the known Lavrent'ev–Bitsadze equation. Here, as distinct from the mentioned papers, we propose another approach, which allows us to obtain an existence theorem with weaker requirements to boundary functions. By the method of spectral decomposition we establish the necessary and sufficient conditions for the unique solvability of problem (2)–(7). We construct a solution to the problem as the sum of the series of eigenfunctions of the corresponding one-dimensional spectral problem. We prove the stability of the solution with respect to boundary data in norms of spaces W_2^n and $C(\overline{D}_\pm)$.

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