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# STABLE APPROXIMATION OF SOLUTIONS OF NONSMOOTH OPERATOR EQUATIONS

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Let $H_1$, $H_2$ be Hilbert spaces. The symbol $\|\cdot\|$ stands for the norm of an element in the corresponding space.

**1.** Consider the operator equation

$$F(x) = 0, \quad x \in H_1, \tag{1}$$

where the nonlinear operator $F : H_1 \to H_2$ has the form $F(x) = F_1(x) + F_2(x)$, the operator $F_1(x)$ is doubly differentiable in the Gateaux sense, and the desired solution $x^*$ of equation (1) satisfies the conditions

$$\|F_1'(x)\| \le N_1, \quad \|F_1''(x)\| \le N_2 \quad \forall x \in \Omega_R(x^*). \tag{2}$$

Here $\Omega_R(x) = \{y \in H_1 : \|y - x\| \le R\}, R > 0$. Assume that the operator $F_2(x)$ satisfies the Lipschitz condition on $\Omega_R(x^*)$:

$$\|F_2(x) - F_2(y)\| \le L\|x - y\| \quad \forall x, y \in \Omega_R(x^*). \tag{3}$$

Because of the nonsmooth term $F_2(x)$ in (1) one cannot apply to this equation the numerical solution methods which are developed for equations with differentiable operators (see, for example, [1]–[3]). However, in a number of cases these methods admit intrinsic modification for equations (1), where the smooth term $F_1(x)$ dominates, in a sense, the nonsmooth component $F_2(x)$. As an example, we mention the nonsmooth version of the Newton–Kantorovich method

$$x_{n+1} = x_n - F_1'(x_n)^{-1}F(x_n). \tag{4}$$

The iterations of (4) strongly converge to $x^*$ if the Lipschitz constant $L$ is sufficiently small and the derivative $F_1'(x)$ is continuously reversible (see [1], p. 150; [4], [5]). From the practical point of view, the requirement of the continuous reversibility of the derivative $F_1'(x)$ or the operator $F_1'^*(x)F_1'(x)$ is often rather restrictive. It does not hold, for instance, in the important for applications case, when $F_1(x)$ is an operator of the Urysohn type (see [6], p. 369) in the Lebesgue or Sobolev spaces. Here $F_1'(x)$ is a linear continuous operator acting from $H_1$ into $H_2$ and in typical cases it is completely continuous for all $x \in H_1$. In this paper, we use the results of the investigation of iteration processes of the gradient type for smooth irregular operators (see [3], Chap. 5) in order to construct a nonsmooth analog of one of these processes. It is necessary for the stable approximation of the solution of equation (1) with an irregular smooth term subject to errors.

**2.** Assume that instead of the operators $F_1$, $F_2$, only their approximations $\widetilde{F}_1, \widetilde{F}_2 : H_1 \to H_2$ are given and for all $x \in \Omega_R(x^*)$,

$$\|\widetilde{F}_1(x) - F_1(x)\| \le \delta, \quad \|\widetilde{F}_1'(x) - F_1'(x)\| \le \delta; \quad \|\widetilde{F}_2(x) - F_2(x)\| \le \delta. \tag{5}$$