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MULTIVALENT FUNCTIONS FROM THE EXTENDED BECKER AND NEHARI CLASSES AND THEIR HYDROMECHANICAL INTERPRETATION

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In the present article we prove that n-valent functions with an arbitrary n belong to the extended Becker class of analytic functions $f(z)$ with the condition

$$2\,\mathrm{Re}\, z|f''(z)/f'(z)| \le 1+\varepsilon, \quad \varepsilon > 0, \tag{1}$$

in the half-plane $H = \{z : \mathrm{Re}\, z > 0\}$. Construction of those functions is based on a power transformation of the form $(w-a)^\alpha$ and on the superposition $T^{-1} f_0 T(z) = f_0(Rz)/R$, which contains an extremal function $f_0(z)$ from [1] and a linear function $T(z) = Rz$. In addition, we demonstrate the relation between $f_0(z)$ and the complex potential of translational flow of an ideal fluid with a vortex at one point (Section 1). Moreover, we clarify the kernel convergence of families of finitely-valent and infinitely-valent domains, which characterize the extension of the Nehari class (see [2]) in the form $(2\,\mathrm{Re}\, z)^2|\{f,z\}| \le 2+\varepsilon$, $z \in H$, where $\{f,z\}$ is the Schwarzian. We give the hydrodynamical interpretation of the corresponding schlicht functions in the form of complex potentials of flows with discontinuities of velocities on boundary streams (Section 2). In Section 3 we cite the characteristic of flows flowing around non-schlicht profiles, which arise in an extension of the Becker and Nehari class in the exterior of the unit disk.

1. Let us start with an auxiliary flow in the plane ω, which is formed by a vortex in translational flow of an ideal incompressible fluid with $\vec{V}(\infty) = V > 0$. The complex potential of such a flow has the form $\Omega = V\omega - \frac{\Gamma}{2\pi i}\ln(\omega - i)$. The critical point is

$$\omega_0 = -i\left(\frac{\Gamma}{2\pi V} - 1\right) \quad (\vec{V}(\omega_0) = 0 \Longrightarrow 2\pi i V(\omega_0 - i) = \Gamma).$$

The simple case is when the velocity of non-perturbed flow $V = 1$ and the intensity of vortex $\Gamma = 2\pi$. Then $\Omega = \omega + i\ln(\omega - i)$. The picture of the streamlines of this current is represented in [3] (p. 372, Fig. 252).

By the change of variables $\Omega = i(z + i\pi/2)$ and $\omega = if(z)$, we arrive at the relation $iz - \pi/2 = if(z) + i\ln i + i\ln(f(z) - 1)$, whence

$$z = f(z) + \ln(f(z) - 1), \quad -\pi < \arg(f(z) - 1) < \pi, \quad f(2) = 2. \tag{2}$$

A function satisfying equation (2) was denoted in [1] by $f_0(z)$. This function carries a half-plane $H = \{z : \mathrm{Re}\, z > 0\}$ to the domain D_{f_0} with the boundary passing through 0 which is the critical point of the flow (see Fig. 1 on p. 4). In [1] it was shown that the function $f_0(z)$ in the half-plane H satisfies the Becker condition

$$|f_0''(z)/f_0'(z)| \le 1/(2\,\mathrm{Re}\, z),$$