ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 7, pp. 68–80. © Allerton Press, Inc., 2011.
Original Russian Text © A.A. Shcheglova, 2011, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2011, No. 7, pp. 80–93.

The Continuous Dependence of Solutions to Algebraic Differential Systems on the Initial Data

A. A. Shcheglova[1*]

[1] *Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences, ul. Lermontova 134, Irkutsk, 664033 Russia*

Received March 29, 2010

Abstract—We consider a nonlinear system of ordinary differential equations which is unsolved with respect to the derivative of the desired vector function and identically degenerate in the definition domain. We study the consistency manifold under assumptions that guarantee the existence of a solution. We prove an analog of the theorem on the continuous dependence of solutions on the initial data, assuming that the latter belong to the consistency manifold.

DOI: 10.3103/S1066369X11070097

Keywords and phrases: *continuous dependence on initial data, consistent initial data, nonlinear differential algebraic equations, existence of a solution.*

1. INTRODUCTION

We consider the following system of nonlinear ordinary differential equations:

$$F(t, x(t), x'(t)) = 0, \;\; t \in T = [0, +\infty), \tag{1.1}$$

where the n-dimensional vector function $F(t, x, y)$ is defined in the domain

$$\mathscr{D} = \{(t, x, y) : t \in T, \; \|x - \overline{x}\| < K_0, \; \|y - \overline{y}\| < K_1\} \subset \mathbb{R}^{2n+1};$$

$x(t)$ is the desired n-dimensional vector function; $\overline{x}, \overline{y} \in \mathbb{R}^n$ are fixed values; K_0 and K_1 are positive constants. Hereinafter we use the following denotations: $\| * \|$ is one of norms in the Euclidian space, $\phi'(t) = \frac{d}{dt}\phi(t)$ and $\phi^{(i)}(t) = \left(\frac{d}{dt}\right)^i \phi(t) \;\; \forall \phi(t) \in \mathbb{C}^i(T)$.

We assume that $F(t, x, y)$ has in $\mathscr{D}$ a sufficient number of continuous partial derivatives with respect to each of its arguments and $\det \frac{\partial F(t,x,y)}{\partial y} \equiv 0$ in $\mathscr{D}$. In particular, such systems are said to be algebraic differential (ADS). The measure of unsolvability of ADS with respect to the derivative of the desired vector function is an integer number r, $0 \le r \le n$, which is called the index.

In the theory of ADS one uses different definitions of the index: the differential index [1], the unsolvability index [2, 3], the tractability index [4–6], and the strangeness index [7]. Different definitions of the index are discussed, for example, in [2, 3]. In this paper we understand the index as the order of a differential operator that transforms (1.1) to a system, where the “differential” and “algebraic” parts are separated.

Let us choose $t_0 \in T$ and formulate the Cauchy problem for system (1.1)

$$x(t_0) = x_0, \tag{1.2}$$

where x_0 is some vector in $\mathbb{R}^n$.

One of characteristic features of ADS is the absence (in the general case) of a continuous dependence of solutions on the initial data. Even in a linear system an arbitrarily small disturbance of the initial data can make the perturbed problem unsolvable in the space $\mathbb{C}^1(T)$.

*E-mail: shchegl@icc.ru.