Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 47, No. 3, pp.28–30, 2003 *UDC 517.4*

MANIFOLDS OVER ALGEBRA OF DUAL NUMBERS, WHOSE CANONICAL FOLIATION HAS EVERYWHERE DENSE LEAF

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1. The Lie algebra L of holomorphic vector fields. Ideals of L

Let us recall the basic definitions of the theory of manifolds over algebras[1] (see [1], [2]). A manifold M is called a manifold over the algebra of dual numbers $R(\varepsilon)$ if M is modelled on a Cartesian power $(R(\varepsilon))^n$ of $R(\varepsilon)$, and the transition mappings are $R(\varepsilon)$-holomorphic (a mapping $f : (U \subset (R(\varepsilon))^k) \to (R(\varepsilon))^m$, where U is an open subset in $(R(\varepsilon))^k$, is said to be $R(\varepsilon)$-holomorphic if its derivative f' is $R(\varepsilon)$-linear). In addition, we will assume that M considered as a real manifold, is a C^∞-differentiable manifold. The number n is called the dimension of M over $R(\varepsilon)$, and the dimension of M over R is $2n$. In every fiber of the tangent bundle TM of M an affinor ε of rank n is determined such that $\varepsilon^2 = 0$. From the definition of a manifold over an algebra it follows that M admits an atlas over $R(\varepsilon)$ such that the matrices of ε with respect to the charts of this atlas are constant (see [1], [2]).

A holomorphic vector field on M is an $R(\varepsilon)$-holomorphic section $s : M \to TM$. This means that on M an $R(\varepsilon)$-atlas $\{(U_i, \varphi_i)\}_{i\in I}$ ($TU_i \cong U_i \times (R(\varepsilon))^n$) exists such that, for every i, $s \circ \varphi_i^{-1} : \varphi_i(U_i) \to (R(\varepsilon))^n$ is an $R(\varepsilon)$-holomorphic mapping and is C^∞-differentiable over R. Let L be a set of all holomorphic vector fields on M. The following results (Propositions 1.1, 1.2) are known.

Proposition 1.1. *L is the Lie algebra over $R(\varepsilon)$.*

Note that Proposition 1.1 remains valid in a more general situation; namely, for a manifold over an arbitrary local algebra (see [2]).

We can indicate another way to describe the Lie algebra L. We will call ε the affinor of dual structure by analogy with the complex structure. An infinitesimal automorphism of dual structure on M is a vector field X on M such that $\mathcal{L}_X(\varepsilon) = 0$, where $\mathcal{L}_X$ is the Lie derivative with respect to X. Then, using the properties of the Lie derivative (see [3], p. 37), we obtain the equality

$$[X, \varepsilon Y] = (\mathcal{L}_X(\varepsilon))Y + \varepsilon[X, Y],$$

where X and Y are arbitrary vector fields on M. Thus, a vector field X is an automorphism of the dual structure ε if and only if $[X, \varepsilon Y] = \varepsilon[X, Y]$ for any vector field Y.

Proposition 1.2. *The set of infinitesimal automorphisms of dual structure coincides with the Lie algebra L.*

[1] Translator's remark: sometimes the term "varieties over alegbras" is also used.