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A Criterion for the Uniform Convergence of Sinc-Approximations on a Segment

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Abstract—In this paper we obtain a criterion for the uniform convergence inside the interval $(0, \pi)$ of values of E. T. Whittaker operators

$$L_n(f,x) = \sum_{k=0}^{n} \frac{\sin(nx - k\pi)}{nx - k\pi} f\left(\frac{k\pi}{n}\right)$$

for continuous functions. This criterion is similar to that of A. A. Privalov for the convergence of interpolation Lagrange–Chebyshev polynomials and trigonometric ones.

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Key words and phrases: *cardinal function, sinc approximation, Lagrange interpolation, convergence, criterion of the uniform convergence, interpolation process.*

The sinc-approximations were introduced in works of G. Plana [1]. Later E. Borel [2] and E. T. Whittaker [3] offered concepts of a cardinal function and a truncated cardinal function. These functions have the following restrictions to the segment $[0, \pi]$:

$$L_n(f,x) = \sum_{k=0}^{n} \frac{\sin(nx - k\pi)}{nx - k\pi} f\left(\frac{k\pi}{n}\right) = \sum_{k=0}^{n} \frac{(-1)^k \sin nx}{nx - k\pi} f\left(\frac{k\pi}{n}\right) = \sum_{k=0}^{n} l_{k,n}(x) f\left(\frac{k\pi}{n}\right). \qquad (1)$$

By now the problem of sinc-approximations is studied in sufficient degree for functions which are analytic on the whole real axis and exponentially decrease at infinity (see, for instance, [4, 5]). One can find the most complete review of relevant results and important applications of sinc-approximations in the book [6]. In addition, several works are connected with the Kotel'nikov theorem [7]; they describe various representations of entire functions by sinc-series with "uniformly distributed" (in a sense) interpolation points (see, for instance, [8–14]). The well-known paper of H. P. Kramer [15] gave rise to investigations of the connection between the so-called "sampling" theorems and the Lagrange interpolation for points from the spectrum of the Sturm–Liouville problem (see, for instance, [16]).

As far as the we know, the approximation by these operators on a segment or on a bounded interval is studied only for certain classes of analytic functions [6, 17] by the reduction to the case of an axis by means of a conformal mapping.

In this paper we establish a criterion for the uniform convergence inside the interval $(0, \pi)$ immediately for operators (1) for continuous functions. This criterion is similar to that of A. A. Privalov for the convergence of Lagrange–Chebyshev interpolation polynomials and trigonometric polynomials ([18–20], [21], pp. 207, 224). It is also analogous to the criterion for the uniform convergence inside the segment $[0, \pi]$ of Lagrange–Sturm–Liouville interpolation processes [22].

Theorem 1. *Let $f \in C([0,\pi])$, $0 \le a < b \le \pi$, $0 < \varepsilon < \frac{b-a}{2}$. For any positive integer n we consider integers p_1, p_2, m_1, and m_2 such that*

$$\frac{\pi(k_1 - 1)}{n} \le a < \frac{\pi k_1}{n}, \quad \frac{\pi k_2}{n} < b \le \frac{\pi(k_2 + 1)}{n};$$

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