ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 10, pp. 65–70. © Allerton Press, Inc., 2013.
Original Russian Text © S.I. Solodushkin, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 10, pp. 77–82.

A Difference Scheme for the Numerical Solution of an Advection Equation with Aftereffect

S. I. Solodushkin[1*]
(Submitted by V.V. Vasin)

[1] *Ural Federal University, ul. Turgeneva 4, Ekaterinburg, 620000 Russia*

Received March 23, 2013

Abstract—We propose a family of grid methods for the numerical solution of an advection equation with a time delay in a general form. The methods are based on the idea of separating the current state and the prehistory function. We prove the convergence of the second-order method coordinatewise and do that of the first-order with respect to time. The proof is based on techniques applied for proving analogous theorems for functional differential equations and on the general theory of difference schemes. We illustrate the obtained results with a test example.

DOI: 10.3103/S1066369X13100095

Keywords and phrases: *advection equations, time delay, difference scheme, numerical methods.*

1. Introduction. The problem. Many papers are dedicated to the qualitative theory of partial functional differential equations (see, for example, [1] and references therein). Since in most cases one cannot solve such equations analytically, the elaboration, substantiation, and computer realization of numerical methods for this class of equations are of essential interest (see the review in [2]). Many difference schemes [3] are known for advection equations without delay. In the paper [4], for an advection equation with a delay, one considers an approximation of the derivative with respect to the phase variable with two nodes, which provides only the first order of convergence with respect to x. The present paper continues the investigation initiated in [5].

Advection equations with delay arise in modeling the dynamics of populations structured with respect to the cell size [6, 7], the age of specimen, etc.

Consider the following advection equation with aftereffect:

$$\frac{\partial u(x,t)}{\partial t} + a\frac{\partial u(x,t)}{\partial x} = f(x,t,u(x,t),u_t(x,\cdot)), \tag{1}$$

where $x \in [0, X]$ and $t \in [t_0;\theta]$ are, respectively, the spatial and time variables (independent ones), $u(x,t)$ is the desired function, $u_t(x,\cdot) = \{u(x,t+\xi),\ -\tau \le \xi < 0\}$ is the prehistory function of the desired function at the moment t, $\tau > 0$ is the value of the delay, and $a > 0$ is a coefficient.

Together with the equation, we state the initial condition

$$u(x,t) = \varphi(x,t),\ \ x \in [0,X],\ \ t \in [t_0-\tau, t_0], \tag{2}$$

the boundary one

$$u(0,t) = g(t),\ \ t \in [t_0,\theta], \tag{3}$$

and the fitting condition

$$g(t_0) = \varphi(0,t_0).$$

We assume that the functional f and functions φ and g are such that problem (1)–(3) has a unique solution. Questions of the existence and uniqueness of a solution to the stated boundary value problem were considered in [1].

[*]E-mail: solodushkin_s@mail.ru.