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The Schwarz Problem for Infinite Sets of Intervals

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Abstract—We solve the Schwarz problem for boundary contours consisting of countable sets of segments with limit point at infinity, including the periodic case. The solution is a result of a reduction to corresponding Riemann boundary-value problems.

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INTRODUCTION

The Schwarz problem for a finite set of intervals is considered in [1] (Chap. IV, pp. 360–363). In the present paper we generalize this theory on the infinite sets of segments by means of the known results concerning the Riemann boundary-value problem on infinite sets of contours (see [2, 3]). In particular, we solve the problem for periodic systems of segments and periodic boundary data [4, 5].

1. STRUCTURE OF A SOLUTION TO THE PROBLEM

1.1. *Statement of the problem.* Let S stand for the complex plane with cuts along segments $L_k = (a_k, b_k)$ $(k = 1, 2, \dots)$ of real axis, $a_1 < b_1 < \dots < a_k < b_k < \dots$ and $\lim\limits_{k\to\infty} a_k = \infty$. We denote $L = \bigcup\limits_{k=1}^{\infty} L_k$.

Find holomorphic in S function

$$\Phi(z) = u(z) + iv(z)$$

with given boundary values of its real part

$$\begin{matrix} u^+(t) = f^+(t), \\ u^-(t) = g^-(t), \end{matrix} \quad t \in L, \tag{1}$$

where given functions $f^+(t) = f_k^+(t)$, $g^-(t) = g_k^-(t)$, $t \in \overline{L}_k$, satisfy the Hölder condition $f_k^+(t), g_k^-(t) \in H_{\lambda_k}(\overline{L}_k)$.

Let us describe structure of a solution to the problem in classes $h(b_k)$, $h(a_k)$, $h(a_k, b_k)$, h_0, $h(c_{k_n})$.

1.2. *Structure of a solution to the problem in the class* $h(b_k)$. The class $h(b_k)$ consists of functions, which are almost bounded near the end-points b_k, and have singularities of order no higher than unit at the rest end-points.

We introduce auxiliary functions

$$\Omega(z) = \frac{\Phi(z) + \overline{\Phi}(z)}{2}, \quad \Psi(z) = \frac{\Phi(z) - \overline{\Phi}(z)}{2},$$

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