Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41. No. 5, pp.60–68, 1997 *UDC 517.9*

INVARIANT SPACES AND DICHOTOMIES OF SOLUTIONS OF A CLASS OF LINEAR EQUATIONS OF THE SOBOLEV TYPE

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Let $\mathfrak{A}$ and $\mathfrak{F}$ be the Banach spaces, an operator L be linear and continuous, i. e., $L \in \mathcal{L}(\mathfrak{A};\mathfrak{F})$, and an operator $M : \operatorname{dom} M \to \mathfrak{F}$ be linear and closed with the domain $\operatorname{dom} M$ dense in $\mathfrak{A}$. Our interest is in invariant subspaces and dichotomies of solutions of the linear operator equation *of Sobolev's type* (see [1])

$$L\dot{u} = Mu. \tag{0.1}$$

If the operator $L^{-1} \in \mathcal{L}(\mathfrak{F};\mathfrak{A})$ exists, then equation (0.1) can be trivially reduced to a pair of equivalent to it standard equations

$$\dot{u} = Su, \qquad \dot{f} = Tf,$$

where $S = L^{-1}M : \operatorname{dom} S \to \mathfrak{A}$, $T = ML^{-1} : \operatorname{dom} T \to \mathfrak{F}$ are linear closed operators with the domains $\operatorname{dom} S = \operatorname{dom} M$ and $\operatorname{dom} T = L[\operatorname{dom} M]$ which are dense in the spaces $\mathfrak{A}$ and $\mathfrak{F}$, respectively. If the operator S (or operator T) is also *sectorial* (see [2], item 1.3), then the given problem has a classical solution (see [3], item 42).

However, the case where the operator L is irreversible, is of greater interest, in particular, when $\ker L \neq \{0\}$. Namely to this case there can be reduced certain collection of problems which had arisen a short time ago in applications (see [4]–[6]). Various particular cases of this problem have already attracted an attention of mathematicians (see [7] and [8]), but only the approach developed recently (see [9]) allows to consider the problem with appropriate completeness.

In Section 1 of the present article we introduce relative spectrum (L-spectrum) of the operator M and prove a theorem on splitting of the spaces $\mathfrak{A}$ and F in correspondence with the splitting of the L-spectrum. These results possess a separate value since they generalize some particular cases (see [9]). In Section 2, some results on the theory of p-sectorial operators and analytical semigroups of operators with kernels (see [9]) are exposed and refined. In Section 3 theorems on existence of invariant spaces and dichotomy of solutions of equation (0.1) (more exactly, of a pair of equivalent to it equations defined on the spaces $\mathfrak{A}$ and $\mathfrak{F}$, respectively) are proved. In Section 4 we give an example which is of interest for applications.

Let us agree on the following: all investigations are carried out in the real Banach spaces, however, in considering "spectral" questions we shall introduce their natural compactification; all the contours are assumed to be oriented by a "counter-clockwise" movement and to bound domains, which lie at the left under such a moving.

Supported by "Universities of Russia" program in the direction: "Fundamental problems of Mathematics and Mechanics", project No. 1.5.16, in part — by State Fellowship and International Scientific Foundation by G. Soros.