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On a Problem with Shift for Degenerate Equation of Mixed Type

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Abstract—In this paper, for a class of mixed type equations we consider a problem with shift on a boundary characteristic. We prove theorems of uniqueness and existence of a solution to formulated problem.

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1. **Definition of Problem A.** Let us consider the equation

$$(\operatorname{sgn} y)|y|^m u_{xx} + u_{yy} - (m/2y)u_y = 0, \;\; m = \text{const} > 0. \tag{1}$$

Let Ω be a finite simply connected domain of a complex plane $z = x + iy$, which is bounded with $y > 0$ by a normal curve $\sigma_0 : x^2 + 4(m+2)^{-2}y^{m+2} = 1$ with ends at points $A(-1, 0)$, $B(1, 0)$, and with $y < 0$ by characteristics AC and BC of Eq. (1), where $C(0, -((m+2)/2)^{2/(m+2)})$. We denote by Ω^+ and Ω^- parts of the domain Ω, which lie, respectively, in half-planes $y > 0$ and $y < 0$, and by C_0 a point of intersection of the characteristic AC with the characteristic, outgoing from a point $E(c, 0)$, where $c \in I = (-1, 1)$ is an interval of the axis $y = 0$. Let $p(x) = \delta - kx$ be a linear diffeomorphism from a set of points of a segment $[-1, c]$ to a set of point of a segment $[c, 1]$, and $p(-1) = 1, p(c) = c$, where $k = (1 - c)/(1 + c), \delta = 2c/(1 + c)$.

Problem A. *In the domain* Ω *it is required to find a function* $u(x, y) \in C\left(\overline{\Omega}\right)$, *which satisfies the following conditions* :

1) *the function* $u(x, y)$ *belongs to the class* $C^2(\Omega^+)$ *and satisfies Eq.* (1) *in this domain*;

2) *the function* $u(x, y)$ *is a generalized solution of class* R_1 ([1], P. 129) *to Eq.* (1) *in the domain* Ω^-;

3) *on the interval of degeneration* AB *the conjugation condition takes place*

$$\lim_{y\to -0} (-y)^{-m/2}\frac{\partial u}{\partial y} = \lim_{y\to +0} y^{-m/2}\frac{\partial u}{\partial y}, \quad x \in I \setminus \{c\}, \tag{2}$$

and these limits can have singularities of order less than unit with $x \to \pm 1$, $x \to c$;

4) *the conditions are fulfilled*

$$u(x, y)|_{\sigma_0} = \varphi(x), \;\; x \in \overline{I}, \tag{3}$$

$$u[\theta(x)] - u[\theta(p(x))] = qu(x, 0) + \psi(x), \;\; x \in [-1, c], \tag{4}$$

$$u(p(x), 0) - u(x, 0) = f(x), \;\; x \in [-1, c], \tag{5}$$

where

$$\theta(x_0) = \frac{x_0 - 1}{2} - i\left[\frac{m+2}{4}(1 + x_0)\right]^{2/(m+2)}$$

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