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APPROXIMATION OF FUNCTIONS IN L_2-METRIC WITH THE JACOBI WEIGHT

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In the article we study the structural properties of the class of functions $f(x_1,x_2)$ with a given order of the best approximation by a two-dimensional angle or rectangle in the metric of $L_{2,\lambda,\delta}$. The function $f(x_1,x_2)\in L_{2,\lambda,\delta}$, provided that $f(x_1,x_2)$ is measurable on $[-1;1]^2$ and $\|f\| = \Big(\int\limits_{-1}^{1}\int\limits_{-1}^{1} f^2(x_1,x_2)\lambda(x_1)\delta(x_2)dx_1dx_2\Big)^{1/2} < \infty$.

Consider weight functions $\lambda(x_1)\delta(x_2)$ of the Jacobi type, where $\lambda(x_1)=(1-x_1)^{\alpha_1}(1+x_1)^{\beta_1}$, $\delta(x_2)=(1-x_2)^{\alpha_2}(1+x_2)^{\beta_2}$, $\alpha_i\ge\beta_i>-1$, $i=1,2$. Let $\{\widehat{\mathrm{L}}_{kl}(x_1,x_2)\}_{k,l=0}^{\infty}$ be an orthonormal system of polynomials $\widehat{\mathrm{L}}_{kl}(x_1,x_2)=\widehat{P}_k^{(\alpha_1,\beta_1)}(x_1)\cdot\widehat{P}_l^{(\alpha_2,\beta_2)}(x_2)$. Here $\{\widehat{P}_k^{(\alpha_1,\beta_1)}(x_1)\}_{k=0}^{\infty}$ is the system of Jacobi polynomials with the parameters α_1, β_1, orthonormal on $[-1;1]$ with the weight $\lambda(x_1)$. Similarly, $\{\widehat{P}_l^{(\alpha_2,\beta_2)}(x_2)\}_{l=0}^{\infty}$ is a system of Jacobi polynomials, orthonormal on $[-1;1]$ with the weight $\delta(x_2)$. Denote by $P_k^{(\alpha,\beta)}$ the standard Jacobi polynomials with the parameters α, β, i.e., the Jacobi polynomials normalized by

$$P_k^{(\alpha,\beta)}(1)=\binom{k+\alpha}{k}.$$

In the space $L_{2,\lambda,\delta}$ define the generalized shift operators. Let $f(x_1,x_2)\sim\sum\limits_{k,l=0}^{\infty}c_{kl}(f)\widehat{\mathrm{L}}_{kl}(x_1,x_2)$ be a Fourier expansion of f by the system $\{\widehat{\mathrm{L}}_{kl}(x_1,x_2)\}$. Give a sequence of continuous on $[0,1/4]$ functions $\{\beta_k^{[\lambda]}(h_1)\}_{k=0}^{\infty}$ such that the series $\sum\limits_{k=0}^{\infty}\beta_k^{[\lambda]}(h_1)c_{kl}(f)\widehat{\mathrm{L}}_{kl}(x_1,x_2)$ converges in $L_{2,\lambda,\delta}$. We denote by $T_{h_1}^{[\lambda]}f$ the sum of the series (in the $L_{2,\lambda,\delta}$ sense) and call the function a *generalized shift operator* (or simply *shift*) of the function $f(x_1,x_2)$ with respect to its first argument by $h_1\in[0,1/4]$, i.e.,

$$T_{h_1}^{[\lambda]}f(x_1,x_2)=\sum_{k=0}^{\infty}\beta_k^{[\lambda]}(h_1)c_{kl}(f)\widehat{\mathrm{L}}_{kl}(x_1,x_2) \tag{1}$$

(the equality is understood in the sense of $L_{2,\lambda,\delta}$). Functional sequences $\{\beta_k^{[\lambda]}(h_1)\}_{k=0}^{\infty}$ are called *coefficients* of the generalized shift operator.

Consider functional sequences $\beta_k^{[\lambda]}$, determined by one of the following methods. If $\lambda(x_1)$ is the Jacobi weight with $\alpha\ge\beta$, $\alpha\ge-1/2$, $\beta>-1$, then the coefficient sequence of a generalized shift can be given as follows:

$$\beta_k^{[\lambda]}(h_1)=(P_k^{(\alpha,\beta)}(1))^{-1}P_k^{(\alpha,\beta)}(\cos h_1). \tag{I}$$