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ITERATIVE METHOD OF ANALYSIS FOR NONLINEAR SINGULARLY PERTURBED INITIAL AND BOUNDARY VALUE PROBLEMS

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In the present article we prove theorems on the existence of a unique and bounded solution for a class of regular and singularly perturbed nonlinear initial and boundary value problems (which are considered from the unified point of view by means of their reduction to a quasilinear system and then to a single-type equivalent integral equation). This complements or refines results known earlier (see [1]–[4]). By means of special matrix transformations (see [5], [6]) we construct via the iterative method an asymptotic decomposition of the solution of singularly perturbed multipoint boundary value problem with weak nonlinearity; this solution is valid as in the absence of resonance correlations, so in the presence of both identical and nonidentical resonances (also in the critical case where spectrum's points may be tangent to the imaginary axis). Note that the new algorithm, convenient for numerical realization, distinguishes the singularities of the solution in a closed analytical form, thus enabling to analyze the singularly perturbed problems from both the quantitative and qualitative standpoints.

In the study in R^n of regular boundary value problems of the form

$$\frac{dy}{dx} = G(y,x); \quad \sum_{j=1}^{n} F_j y(x_j) = \alpha; \quad (x \in [a,b]; \quad a = x_1 < \cdots < x_n = b), \tag{1}$$

where F_j are some constant $n \times n$ matrices ($j = \overline{1,n}$), we emphasize the question on the existence of their solution in a certain neighborhood of a solution $\varphi(x)$ of the initial system $\varphi' \equiv G(\varphi, x)$. In this case, if the function $G(y,x)$ is sufficiently smooth, problem (1) can be transformed (after respective change of variables) to the quasilinear problem

$$\frac{dz}{dx} = A(x)z + f(z,x); \quad \sum_{1}^{n} F_j z(x_j) = \beta; \quad (A(x) = G_y(\varphi(x),x) \in C[a,b]; \quad f(0,x) \equiv 0; \quad |z| < k_0) \tag{2}$$

and then to an equivalent integral equation (see [5])

$$z(x) = \Phi(x)C + \sum_{1}^{n} \Phi_k(x) \int_{x_k}^{x} \Phi^{-1}(t) f(z,t)dt \equiv L(z), \tag{3}$$

where

$$C = F^{-1}\Big(\beta - \sum_{j=1}^{n} F_j \sum_{k=1}^{n} \Phi_k(x_j) \int_{x_k}^{x_j} \Phi^{-1}(t) f(z,t)dt\Big); \quad F = \sum_{1}^{n} F_j \Phi(x_j)$$

(equivalence between problem (2) and equation (3) can be verified (see [5]) by direct differentiation of equation (3) in case $\det F \neq 0$); $\Phi(x) = \sum_{k=1}^{n} \Phi_k(x) = \{\varphi_{jq}(x)\}_1^n$ is the fundamental matrix of the