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Absolute Convergence of Double Series of Fourier–Haar Coefficients for Functions of Bounded p-Variation

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Abstract—We consider functions of two variables of bounded p-variation of the Hardy type on the unit square. For these functions we obtain a sufficient condition for the absolute convergence of series of positive powers of Fourier coefficients with power-type weights with respect to the double Haar system. This condition implies those for the absolute convergence of series of Fourier–Haar coefficients of one-variable functions which have a bounded Wiener p-variation or belong to the class $\mathrm{Lip}\,\alpha$. We show that the obtained results are unimprovable. We also formulate N-dimensional analogs of the main result and its corollaries.

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1. INTRODUCTION

An orthonormal on the segment $[0,1]$ and complete in the space $L[0,1]$ system of functions $\{\chi_n\}_{n=1}^{\infty}$ was constructed by A. Haar [1, 2] in 1909. Let us recall the definition of this system. Let $\chi_1(x) \equiv 1$ on $[0,1]$. Introduce the denotation $I_i^k = \left(\frac{i-1}{2^k}, \frac{i}{2^k}\right)$, $i = 1, \dots, 2^k$, $k = 0, 1, \dots$ We represent a positive integer $n \geq 2$ in the form $n = 2^k + i$, $i = 1, \dots, 2^k$, $k = 0, 1, \dots$, and put $\chi_n(x) = \sqrt{2^k}$ for $x \in I_{2i-1}^{k+1}$, $\chi_n(x) = -\sqrt{2^k}$ for $x \in I_{2i}^{k+1}$, and $\chi_n(x) = 0$ for $x \in [0,1] \setminus \overline{I}_i^k$, where $\overline{I}_i^k$ is the closure of the interval I_i^k. At the interior discontinuity points we equate the Haar functions to halves of sums of their left and right limits, and at the endpoints of the segment $[0,1]$ we do them to their limit values from within the segment.

Using the constructed system, A. Haar has answered affirmatively to the following question stated by D. Hilbert: Whether there exists an orthonormal system such that the Fourier series of continuous functions in this system uniformly converge to these functions on the segment of orthogonality. The functions of the Haar system are step ones and, consequently, this system is not a basis in the space $C[0,1]$ of continuous functions. But, as was shown by G. Faber [3] in 1910, any continuous on the segment $[0,1]$ function is uniquely expandable into a series which uniformly converges to it in the system of functions $\left\{1, \int_0^x \chi_n(t)dt\right\}_{n=1}^{\infty}$. Thus, he constructed a basis in the space $C[0,1]$ seventeen years earlier than J. Schauder [4], who is often considered to be the first who obtained a basis in the space $C[0,1]$. The most "popular" Schauder basis in $C[0,1]$ was obtained by a normalization of the G. Faber one. J. Schauder [5] has proved that the Haar system is a basis in the space $L^p[0,1]$, $1 \leq p < \infty$.

The main results on the Haar series can be found in books [6, 7] and in our review [8].

The absolute convergence of series of Fourier–Haar coefficients was first studied by Z. Ciesielski and J. Musielak [9]. They have proved the following theorem: If $f \in V[0,1] \bigcap \mathrm{Lip}\,\alpha$ $(0 < \alpha \leq 1)$, then $\sum_{n=1}^{\infty} |\widehat{f}(n)| < \infty$, where $\widehat{f}(n) = \int_0^1 f(x)\chi_n(x)dx$ are Fourier–Haar coefficients of the function f, $V[0,1]$

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