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Orthogonality of Eigenwaves in a Semi-Open Elastic Waveguide

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Abstract—We consider a two-layer elastic waveguide structure such that its one layer is unbounded in the lateral direction, and the standard boundary conditions are stated on the boundary of another one. We study several kinds of eigenwaves for this structure. In general, they have a complex-valued longitudinal propagation constant (the spectral parameter). On the base of the Green formula we introduce the scalar product of two waves and prove that the system of eigenwaves of the semi-open elastic waveguide is orthogonal. We construct families of waves which belong to discrete and continuous parts of the spectrum.

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Brief communication

The stratified structures have waveguide properties, namely, they admit the sourceless propagation of waves. In [1] I.N.Pleshchinskii and N.B.Pleshchinskii, using the method of separation of variables, obtain eigenwaves of a semi-open dielectric waveguide related to the discrete and continuous parts of the spectrum. They prove that the set of values of the longitudinal propagation constant (the spectral parameter) on the complex plane consists of the vertical semiaxis, a horizontal segment, and isolated points ("a hockey stick with pucks"). They also prove that the eigenwaves of a semi-open waveguide are orthogonal in the sense of the scalar product (introduced in an intrinsic way) and form a complete system of modes. Any wave that is propagating in an infinite semi-open waveguide can be expanded in this system as in a basis.

In this paper, in a two-dimensional (plane) case we study properties of eigenwaves of a waveguide structure formed by an elastic strip and an elastic half-plane which are in full contact (see the figure).

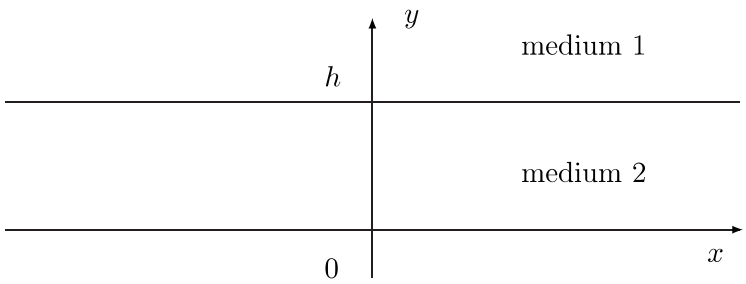

Figure. A semi-open elastic waveguide.

The elastic waves in stratified media are described, for instance, in monographs [2] and [3], where the emphasis is placed on the reflection and refraction problems for elastic waves and on the physical interpretation of results. The eigenwaves that propagate in an elastic half-plane (the so-called Rayleigh surface waves) and the Love waves that propagate in an elastic layer which lies on an elastic half-space are well-known.

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