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ON THE EXISTENCE AND UNIQUENESS OF SOLUTION AND ON THE CONVERGENCE RATE OF THE BUBNOV–GALYORKIN METHOD FOR A QUASILINEAR EVOLUTIONARY PROBLEM IN A HILBERT SPACE

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1. Introduction

In the present article the Cauchy problem for the following quasilinear evolutionary equation of second order is considered:

$$Q[u,t] \equiv u''(t) + K(t)u'(t) + A(t)u(t) + B(t,u(t)) = 0, \quad t \in [0,T], \tag{1.1}$$

$$u(0) = u_0^{(0)}, \quad u'(0) = u_1^{(0)}. \tag{1.2}$$

Here u is the sought-for function which maps the segment $[0,T]$ of the numerical axis $\mathbb{R}$ to a real separable Hilbert space H; $K(t)$, $A(t)$ $(t \in [0,T])$ are linear operators acting in H, moreover, all the operators $A(t)$ are selfadjoint and positive definite; B is a nonlinear operator; $u_0^{(0)}$, $u_1^{(0)}$ are given elements of the space H. Conditions which should be imposed upon the operator coefficients and initial data of problem (1.1), (1.2) will be formulated in more detail in Section 2. The class of problems, for which these conditions are fulfilled, includes, in particular, some initial-boundary value problems of the Mathematical Physics, which are obtained via the variational Hamilton–Ostrogradskiĭ principle.

The main objective of the present article is to obtain for problem (1.1), (1.2) an a priori estimate of the Bubnov–Galyorkin method (BGM) under a special choice of a basis (in the capacity of basis elements of BGM we shall use eigen-elements of an auxiliary operator which is similar to (see [1], p. 21) and makes an acute angle (see [2]) with each of the operators $A(t)$). Earlier, estimates of that sort were obtained for evolutionary problems which contain the second derivative of the desired function with respect to time in [3]–[5], and for stationary problems and evolutionary problems of first order with respect to time — in a series of works, among which we refer to [6]–[9].

Along with an estimate of the error of BGM for problem (1.1), (1.2), we shall establish a result concerning unique resolvability. This result is necessary for our exposition to be both complete and correct. In the known literature, results of that kind for the problem under consideration and even for problems of more general form are absent. For nonlinear problems in an abstract Hilbert space, which are similar to the problem under consideration, the result on the existence of a generalized solution was obtained in [10], while the result on stability (which implies solution's uniqueness) can be found in [11].

Let us note that the method of proof of solution's existence, which is adopted in the present article, differs from the known method of compactness, which is traditionally used along with BGM in the investigation of problems similar to (1.1), (1.2) (see, e. g., [10], and also [12], Chap. I). Namely, in the present article we establish an a priori estimate for the difference between any