Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 3, pp. 74–77, 1998 *UDC 517.956*

MULTIVARIATE CENTRAL WAVES OF QUASILINEAR SYSTEMS

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1. *Introduction.* The present article is devoted to solving a multivariate Goursat problem on central wave. The univariate case of the problem was considered in [1] (p. 538) – [5]. We establish sufficient conditions for the existence and uniqueness of a holomorphic solution of the Goursat problem for systems of quasilinear equations of the first order on an analytical manifold. The problem will be solved in a general situation.

2. *Local problem.* On a complex m-dimensional analytical surface with local coordinates x_1, x_2, $x = (x_3, x_4, \dots, x_m)$ we define the differential equations

$$\begin{gathered}\sum_{k=1}^{m} A^k(x_1, x_2, x, W)(\mathfrak{W}, \mathfrak{B})_{x_k} = f(x_1, x_2, x, W),\\ \sum_{k=1}^{m} C^k(x_1, x_2, x, W)\mathfrak{U}_{x_k} = h(x_1, x_2, x, W),\end{gathered} \tag{1}$$

$$W = (w_1, w_2, \dots, w_n) = (\mathfrak{W}, \mathfrak{B}, \mathfrak{U}),$$
$$\mathfrak{W} = (w_1, w_2, \dots, w_q), \quad \mathfrak{B} = (w_{q+1}, w_{q+2}, \dots, w_p), \quad \mathfrak{U} = (w_{p+1}, w_{p+2}, \dots, w_n),$$
$$f = (f_1, f_2, \dots, f_p), \quad h = (f_{p+1}, f_{p+2}, \dots, f_n);$$
$$A^k = (a^k_{ij}) \ \ (1 \le i, j \le p) : a^1_{ij} = 0 \ \ (i \ne j), \quad a^1_{ii} = 1 \ \ (1 \le i \le q), \quad a^1_{ii} = 0 \ \ (q+1 \le i \le p),$$
$$a^2_{ij} = 0 \ \ (i \ne j, \ \ q+1 \le i \le p, \ \ 1 \le j \le p), \quad a^2_{ii} = 1 \ \ (q+1 \le i \le p);$$
$$C^k = (a^k_{ij}) \ \ (p+1 \le i, j \le n) : a^2_{ij} = 0 \ \ (i \ne j), \quad a^2_{ii} = 1 \ \ (p+1 \le i \le n).$$

The coefficients of system (1) are defined in the domain $S \subset \mathbb{C}^{m+n}$ with the projection $s \subset S$ in $\mathbb{C}^m$. We write $s_3 = \{(x_1, x_2, x) \in s : x_2 = 0\}$, $s_0 = \{(x_1, x) \in s_3 : x_1 = 0\}$, $s_1 = s_3 \setminus s_0$.

The quantity s_3 is considered to be a characteristic of the family Γ_1 of system (1). It is the range of the function $\Phi(x_1, x)$ ($\Phi = (\varphi_1, \varphi_2, \dots, \varphi_n)$).

Let us state the following problem: We seek in s_* ($s_1 \subset s_* \subset s \setminus s_0$) for a solution $W(x_1, x_2, x)$ of system (1), which satisfies on s_1 the following condition

$$W(x_1, 0, x) = \Phi(x_1, x) \tag{2}$$

and admits a family of characteristics Γ_1 which contain s_0.

The characteristics of system (1) are determined via the integrals of the following equation (see [6], p. 85):

$$\det \sum_{k=1}^{m} A^k \eta_{x_k} \det C^k \eta_{x_k} = 0.$$

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