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The First-Approximation Stability of a Nonstationary System with Delay

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Abstract—We study the exponential stability of a nonlinear system of differential equations with constant delay such that the right-hand side of one of its subsystems contains the multiplier e^t. We obtain a sufficient condition for the first-approximation stability of this system.

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We consider the following nonlinear system with constant delay:

$$\begin{aligned} dx(t)/dt &= A_1x(t) + A_2x(t-\tau) + B_1y(t) + B_2y(t-\tau) \\ &\quad + f^1(t, x(t), x(t-\tau)) + f^2(t, y(t), y(t-\tau)), \\ dy(t)/dt &= e^t[A_3x(t) + A_4x(t-\tau)x(t-\tau) + B_3y(t) + B_4y(t-\tau) \\ &\quad + f^3(e^t, x(t), x(t-\tau)) + f^4(e^t, y(t), y(t-\tau))], \quad t \geq t_0, \quad \tau = \mathrm{const}, \quad \tau > 0. \end{aligned} \tag{1}$$

Here A_k, B_k $(k = 1, 2, 3, 4)$ are positive $m \times m$ matrices, $x(t)$ and $y(t)$ are m-dimensional vector functions with respect to time (the argument) t, $f^j(t, x(t), x(t-\tau))$ and $f^l(t, y(t), y(t-\tau))$ $(j = 1, 3;$ $l = 2, 4)$ are nonlinear vector functions such that $f^j(t, 0, 0) = 0$ and $f^l(e^t, 0, 0) = 0$, and the following bounds are valid in a neighborhood of the origin of coordinates:

$$\begin{aligned} &\|f^j(t, x(t), x(t-\tau))\| \leq \widehat{\delta}_1(t)[\|x(t)\| + \|x(t-\tau)\|], \quad \int_{t-1}^{t} \widehat{\delta}_1(s)ds \leq \widehat{\varepsilon}_1, \\ &\|f^l(e^t, y(t), y(t-\tau))\| \leq \widehat{\delta}_2(e^t)[\|y(t)\| + \|y(t-\tau)\|], \quad \int_{t-1}^{t} \widehat{\delta}_2(s)ds \leq \widehat{\varepsilon}_2, \end{aligned} \tag{2}$$

where $\widehat{\varepsilon}_1$ and $\widehat{\varepsilon}_2$ are sufficiently small positive values. We define the norm of a vector $w = \{w_j\}^{(\top)}$ (here w_j are components of the vector w) by the equality $\|w\| = \sum_{j=1}^{m} |w_j|$. We define the norm of a matrix $D = \{d_{ij}\}$ $(i, j = 1, \dots, m)$ correspondingly to the norm of a vector ([1], P. 12), namely,

$$\|D\| = \max_j \sum_i |d_{ij}|.$$

Consider the following linear homogeneous "unperturbed" system (the first approximation system):

$$\begin{aligned} dx^0(t)/dt &= A_1x^0(t) + A_2x^0(t-\tau) + B_1y^0(t) + B_2y^0(t-\tau), \\ dy^0(t)/dt &= e^t[A_3x^0(t) + A_4x^0(t-\tau) + B_3y^0(t) + B_4y^0(t-\tau)], \quad t \geq t_0 > 0. \end{aligned} \tag{3}$$

We assume that roots λ of the characteristic equation

$$|A_1 + A_2e^{-\lambda\tau} - \lambda E| = 0 \tag{4}$$

have negative real parts, i.e.,

$$\mathrm{Re}\,\lambda < -\beta_1, \quad \beta_1 = \mathrm{const}, \quad \beta_1 > 0. \tag{5}$$