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Poly-Element Equations Reducible to Moment Problem for Entire Functions of Class A

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Abstract—We consider poly-element linear functional equations in the class of analytic functions in the complex plane with cuts along certain segments of imaginary axis. The obtained results are applied for study of the Stieltjes moment problem for entire functions of exponential type in the class A.

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Introduction. Analytic function $F(z)$ belongs to class A if $\sum\limits_{k=1}^{\infty} |\mathrm{Im}(a_k^{-1})| < \infty$, where a_k are roots of $F(z)$ enumerated in order of increasing of their absolute values. These functions have a number of remarkable properties (see [1], Chap. V). The properties of entire functions of exponential type (e. f. e. t.) of the form

$$F(z) = \int_{-\lambda}^{\lambda} \exp(izx) d\sigma(x), \tag{1}$$

where $\sigma(x)$ is a function of bounded variation ([2], Chap. 1, § 4, Item 3) are studied especially well. They are e. f. e. t. of completely regular growth bounded in modulus on the real axis. Their indicator diagrams are segments of imaginary axis $[-\lambda i, \lambda i]$ if $\sigma(x)$ is not constant in certain neighborhoods of end points, what is assumed below. Let $\Gamma = (-2i, -i) \cup (i, 2i)$, and D be square with vertices $\pm 1 \pm i$.

The paper consists of three items. In Item 1 we investigate poly-element functional equation

$$(Vf)(z) \equiv (V_1, f)(z) - (V_1, f)(iz) = g(z), \;\; z \in D, \tag{2}$$

where $(V_1, f)(z) \equiv f(z+1+2i) + f(z-1+2i) + f(z+1-2i) + f(z-1-2i) + f(z-1) + f(z+1)$, under the following assumptions.

1) We seek solution in class B of functions representable by the Cauchy type integrals

$$f(z) = \frac{1}{2\pi i} \int_{\Gamma} (\tau - z)^{-1} \phi(\tau) d\tau \tag{3}$$

with odd densities $\phi(\tau)$ belonging to the Hölder class $H(\Gamma)$. In other words, this is the class of even functions which are holomorphic in the plane with cut along Γ and vanishing in the point at infinity.

2) The right-hand side $g(z)$ is holomorphic in D, $g(iz) = -g(z)$, and $g^+(t) \in H(\partial D)$.

In Item 2 we solve a lacunary Stieltjes moment problem for e. f. e. t. (1), which are associated in Borel sense (see [3], § 1) with lower function $f(z) \in B$.

In Item 3 we consider a generalization of the obtained results.

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