ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2016, Vol. 60, No. 9, pp. 69–73. © Allerton Press, Inc., 2016.
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On Stabilization of Solutions to Incomplete Second-Order Integrodifferential Equations

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Received February 5, 2016

Abstract—We consider abstract incomplete linear second-order integrodifferential equations in a Hilbert space. Operator coefficients of the equations are unbounded selfadjoint nonnegative operators. These equations arise naturally in viscoelasticity and hydroelasticity. We prove a theorem on asymptotic stability of strong solutions of the equations.

DOI: 10.3103/S1066369X16090097

Keywords: *integrodifferential equation, stability.*

Consider the following Cauchy problem for an incomplete second-order integrodifferential equation in a Hilbert space H:

$$\frac{d^2u(t)}{dt^2} = -Au(t) + \sum_{l=1}^{m} \int_0^t \exp\big(-b_l(t-s)\big) B_l u(s)\, ds + f(t), \quad u(0) = u^0, \;\; u'(0) = u^1. \qquad (1)$$

Here A is a self-adjoint and positive operator and, for every $l = \overline{1,m}$, B_l is a self-adjoint and nonnegative operator: $A = A^* \gg 0$, $B_l = B_l^* \geq 0$. We will assume that b_l, $l = \overline{1,m}$, are positive pairwise distinct and ordered such that $0 < b_1 < b_2 < \cdots < b_m$.

Equations of type (1) arise in viscoelasticity (see, e.g., [1]; [2], P. 143), and hydroelasticity ([3], P. 397; [4]).

Definition. A strong solution to the Cauchy problem (1) is a function $u(t)$ such that $u(t) \in \mathcal{D}(A)$, $u'(t) \in \mathcal{D}(A^{1/2})$ for $t \in \mathbb{R}_+$, $Au(t), A^{1/2}u'(t) \in C(\mathbb{R}_+; H)$, $u(t) \in C^2(\mathbb{R}_+; H)$, the initial conditions from (1) hold and $u(t)$ satisfies the integrodifferential equality from (1) for every $t \in \mathbb{R}_+ := [0; +\infty)$.

Theorem. *Let the following conditions be fulfilled*:

1) $\mathcal{D}(A) \subset \mathcal{D}(B_l)$ *for every* $l = \overline{1,m}$;

2) *there exist* $a \in (0,1)$ *and* $q \in \{1, \ldots, m\}$ *such that*

$$a\|A^{1/2}u\|_H^2 \leq \frac{1}{b_q}\|B_q^{1/2}u\|_H^2 \leq \sum_{l=1}^{m} \frac{1}{b_l}\|B_l^{1/2}u\|_H^2 \leq (1-a)\|A^{1/2}u\|_H^2 \;\; \forall\, u \in \mathcal{D}(A^{1/2}) = \mathcal{D}(B_q^{1/2});$$

3) $u^0 \in \mathcal{D}(A)$, $u^1 \in \mathcal{D}(A^{1/2})$, *and* $f(t) \in C^1(\mathbb{R}_+; H)$.

Then the Cauchy problem (1) *has a unique strong solution. Moreover,*

$$\lim_{t\to+\infty} f(t) = f \implies \lim_{t\to+\infty} u(t) = A^{-1/2}\left[I - \sum_{l=1}^{m} \frac{1}{b_l}\big(B_l^{1/2}A^{-1/2}\big)^*\big(B_l^{1/2}A^{-1/2}\big)\right]^{-1} A^{-1/2}f.$$

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