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On the Virtual Residual p-Finiteness of the Free Product of Polycyclic Groups with Normal Amalgamated Subgroups

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Abstract—Let G be the free product of polycyclic groups A and B with normal amalgamated subgroups H and K. We prove that, for any prime p, the group G is a virtually residually p-finite group.

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1. INTRODUCTION

Let $\mathcal{K}$ be a class of groups. Recall that a group G is said to be residually of class $\mathcal{K}$ (or, briefly, residually $\mathcal{K}$) if, for each non-identity element x of G, there exists a homomorphism of G onto a group from $\mathcal{K}$ such that the image of x does not coincide with the identity. If $\mathcal{F}$ denotes the class of all finite groups, then the notion of a residually $\mathcal{F}$ group coincides with the classical notion of a residually finite group. Along with the property of residual finiteness, that of residual p-finiteness (the property for a group to be residually $\mathcal{F}_p$) is the object of study, where p is a prime and $\mathcal{F}_p$ is the class of all finite p-groups. In this paper, we also consider the property of virtual residual p-finiteness, which is intermediate between the properties of residual finiteness and residual p-finiteness. Recall that a group G is said to be virtually residually p-finite if it contains a residually p-finite subgroup of finite index.

As an example of residually finite group one can take an arbitrary polycyclic group. What is more, any polycyclic group is virtually residually p-finite for each prime p. This classical result was obtained by A. L. Shmel'kin [1].

The free product of two residually finite (residually p-finite, virtually residually p-finite) groups is a residually finite (residually p-finite, virtually residually p-finite) group [2, 3].

Now we pass to free products of groups with amalgamated subgroups. Let A and B be groups, H and K subgroups of A and B, respectively, and let φ be an isomorphism of H onto K. Let $G = (A * B;\ H = K,\ \varphi)$ be the free product of A and B with subgroups H and K amalgamated over φ. It is well-known that the groups A and B are naturally embedded into G. By this reason, we will assume in what follows that A and B are subgroups of G. Then $A \cap B = H = K$. In what follows we will denote the group G by $G = (A * B,\ H)$ and call it the free product of the groups A and B with amalgamated subgroup H.

Obviously, in order for the group G to be residually finite (residually p-finite, virtually residually p-finite), it is necessary that the groups A and B be residually finite (residually p-finite, virtually residually p-finite). Simple examples show that the above mentioned conditions are not sufficient.

The most used approach to the study of residual finiteness (residual p-finiteness, virtual residual p-finiteness) of the group G is as follows: Besides the condition of residual finiteness (residual p-finiteness, virtual residual p-finiteness), some additional conditions are imposed upon the groups A and B. As a rule, additional restrictions are also imposed upon the amalgamated subgroup H. Examples of such restrictions are: The finiteness of the subgroup H, its cyclicity, the finiteness of the indices of H in A and B, the normality of H in A and B.

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