Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 4, pp. 98–107, 1997 *UDC 519.633*

THE DECOMPOSITION METHOD FOR SINGULARLY PERTURBED BOUNDARY VALUE PROBLEMS WITH THE LOCAL PERTURBATION OF THE INITIAL CONDITIONS. EQUATIONS WITH CONVECTIVE TERMS

G.I. SHISHKIN and I.V. TSELISHCHEVA

We consider the Dirichlet problem for singularly perturbed parabolic equations in the case of one spatial variable (on a segment). The initial condition of the problem has unit local perturbations of a finite amplitude on a narrow subdomain near $x = 0$ of the width 2δ. The perturbing parameter ε^2, which is the coefficient at highest derivatives of the equation, and the parameter δ may attain arbitrary meanings from the semi-intervals $(0, 1]$ and $(0, d]$, respectively. Here $2d$ is the length of the segment. For $\varepsilon = 0$, the parabolic equation degenerates into a hyperbolic equation of the first order, which contains derivatives with respect to the spatial and time variables. Problems of that kind arise in modeling processes of heat expansion (for $t > t_0 > 0$, $t_0 = \varepsilon^{-2}\delta^2$) in the case of concentrated momentary sources.

It is shown that for problems of that kind, in the case of classical difference approximations, there do not exist rectangular piecewise uniform grids, on which the solution of the difference scheme were converging uniformly with respect to the parameters ε and δ (or, in more short form, (ε, δ)-uniformly). For the above-mentioned boundary value problems, with the use of the method of additive selection of singularities and mobile concentrating (at a vicinity of the transition layer) grids, whose nodes are situated along the characteristic of the limit equation, we construct monotone difference schemes which converge (ε, δ)-uniformly.

Introduction

The solutions of boundary value problems for singularly perturbed equations in the case of smooth initial conditions (when the latter change for a finite value in a narrow domain) possess a bounded smoothness. The derivatives of the solution increase unboundedly when the quantity δ — the semi-width of the domain of drastic change of initial data — and (or) the perturbing parameter ε tend to zero. The presence of the local perturbations leads to appearance of the inner (transitional) layers under small values of the parameters ε and δ. Let us note that the problem is singular even with $\varepsilon = 1$; when the parameter δ tends to zero, the smoothness worsens. This limited smoothness of the solution of the boundary value problem supply difficulties in its numerical solving (see, e.g., [1]–[4]). For small values of the parameters ε and δ the errors of approximate solutions which are obtained by means of classical difference schemes, turn to be commensurable with the sought-for solution. In this connection we state the problem of developing the specific difference schemes whose solutions' error were independent of meanings of parameters ε and δ, i.e., the difference schemes converging (ε, δ)-uniformly.

Supported by the Russian Foundation for Basic Research (code of project 95-01-00039) and partially by the Netherlands Research Organization NWO (grant No. 047.003.017).