ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2016, Vol. 60, No. 1, pp. 68–72. © Allerton Press, Inc., 2016.
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A Method of Bi-Coordinate Variations with Tolerances and Its Convergence

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Received May 23, 2015

Abstract—We propose a method of bi-coordinate variations for optimal resource allocation problems, which involve simplex type constraints. It consists in making coordinate-wise steps together with special threshold control and tolerances whose values reduce sequentially. The method is simpler essentially than the usual gradient ones, which enables one to apply it to large dimensional optimization problems. We establish its convergence and rate of convergence under rather mild assumptions.

DOI: 10.3103/S1066369X16010084

Keywords: *optimization problems, resource allocation, bi-coordinate variations, threshold control, rate of convergence.*

INTRODUCTION

The usual optimization problem consists in finding the minimal value of some goal function $f : X \to \mathbb{R}$ on a feasible set D such that $D \subseteq X \subseteq \mathbb{R}^n$. For brevity, we write this problem as

$$\min_{x\in D} \to f(x), \tag{1}$$

its solution set is denoted by D^* and the optimal value of the function by f^*, i.e., $f^* = \inf_{x\in D} f(x)$.

We will consider a special class of optimization problems, where the function f is supposed to be smooth on X and the set D is defined by simplex type constraints, i.e.,

$$D = \left\{x \in \mathbb{R}^n_+ \mid \langle a, x\rangle = b\right\}, \tag{2}$$

where b is a fixed positive number, a is a fixed vector with positive coordinates, $\langle a, x\rangle$ denotes the usual scalar product of a and x, $\mathbb{R}^n_+$ denotes the non-negative orthant in $\mathbb{R}^n$. Then D is nonempty and compact, hence problem (1)–(2) has a solution.

It is known that many problems of optimal allocation of some resource within a system with n elements reduce to problems of the form (1)–(2) (see, e.g., [1–3] and references therein). Besides, the same optimization formulation is paid a significant attention due to its various big data applications (see, e.g., [4, 5] and references therein). All these problems have great dimensionality, where even calculation of all the components of the gradient may be too hard. For this reason, we are interested in developing special iterative methods, which keep the convergence properties of the usual ones, but reduce the total computational expenses. Observe that these problems do not require high accuracy of solutions. For instance, the usual coordinate descent methods appear rather efficient in the case of unconstrained optimization; (see, e.g., [6]). However, their streamlined extension for the constrained is rather difficult since each single coordinate direction becomes infeasible.

For this reason, some other coordinate-wise methods deserve more attention. Namely, the bi-coordinate descent method for optimization problems of the form (1)–(2) was proposed in [7]. A description of its recent versions can be found in [8]. However, these methods are based on computing

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