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ON POSITIVE RADIAL SYMMETRIC SOLUTION OF DIRICHLET PROBLEM FOR A CERTAIN NONLINEAR EQUATION AND A NUMERICAL METHOD FOR ITS EVALUATION

E.I. Abduragimov

In the unit disk $K = \{(x, y) \in R^2,\ r^2 = x^2 + y^2 < 1\}$ with the boundary Γ we consider the following Dirichlet problem:

$$\Delta u + ar^m u^{2k} = 0, \quad (x, y) \in K, \tag{1}$$

$$u|_\Gamma = 0. \tag{2}$$

Here $m \geq 0$ and $k \geq 1$ are integer numbers, $a \equiv \text{const} > 0$.

By means of S.I. Pokhozhaev's method of spherical stratification (see [1], [2] and others) one can prove the existence of a positive solution $u \in \overset{\circ}{W}{}^1_2(K)$ of problem (1)–(2). The general theory of quasilinear elliptic equations implies the analyticity of this solution for $r < 1$. As it is known (see [4]), for $m = 0$, any positive solution of problem (1)–(2) is radial symmetric. In [3] there was proved the uniqueness of a positive solution of problem (1)–(2) for $m = 0$, $k = 1$.

In the present article we prove the existence of a unique positive solution of problem (1)–(2) in the class $C^2(\overline{K})$ and propose a numerical method for evaluation of this solution.

Note that the uniqueness of a positive solution of the Dirichlet problem was investigated by a number of authors for the equations which are more general than (1), but this was done in a different formulation of the problem and by means of other methods. For example, in [5], the uniqueness of a positive solution of the Dirichlet problem $u|_{\partial\Omega} = +\infty$ for equation $\Delta u = p(x)u^k$ was proved by means of a priori bounds. In [6], there was proved a conditional theorem on the uniqueness of a positive solution of the Dirichlet problem in a ring for similar equations.

1. The existence and uniqueness of a radial symmetric positive solution

Let us show the existence of a radial symmetric solution of problem (1)–(2). In other words, we seek a function $u = u(r)$ from the class $C^2[0, 1]$ satisfying the following two-point boundary value problem:

$$u'' + \frac{u'}{r} + ar^m u^{2k} = 0, \quad 0 < r < 1, \tag{1.1}$$

$$u'(0) = 0, \quad u(1) = 0. \tag{1.2}$$

To this end we use a group of linear transformations which was applied by Z. Na in [7]

$$\begin{aligned} r &= A^{\alpha_1}\overline{r}, \\ u &= A^{\alpha_2}\overline{u}. \end{aligned} \tag{1.3}$$