Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 4, pp. 3–19, 1997 *UDC 517.929*

SEMIDISCRETE METHOD FOR SOLVING EQUATIONS OF A ONE-DIMENSIONAL MOTION OF VISCOUS HEAT-CONDUCTIVE GAS WITH NON-SMOOTH DATA

A.A. AMOSOV and A.A. ZLOTNIK

Systems of quasilinear equations of the motion of a viscous heat-conductive gas (compressible liquid) are related to the basic models of the continuous media mechanics (see [1], [2]). For problems with one spatial variable x the theory of global generalized solutions was exposed in [2] (Chap. 2) for the case of both initial and boundary data from W_2^1. The theory of difference methods for these problems was developed in [3]–[6]. Global generalized solutions in the case of different classes of discontinuous data were studied in [7]–[11] (in [11] in the small sense with respect to the data).

In the present article we construct and study a semidiscrete difference (by the variable x) method for solving inhomogeneous initial-boundary value problems with discontinuous both initial and boundary data. In particular, the initial velocity and temperature are required only to have finite complete energy and entropy, while it suffices that the boundary values of the velocity to have a finite variation. The density of the heat sources and the boundary heat flux can be functions from L_1. The equations' coefficients which define the gas' properties can also be discontinuous with respect to x (the model of an inhomogeneous gas).

The global estimates for approximate solutions and their convergence to the generalized solutions of problems under consideration are proved. Moreover, the proper existence of solutions is established in passing to the limit and represents a new result since it arises under more general conditions than in [9], [10]. We essentially use the technique developed in [2], Chap. 2; [3], [10], [12]. We should note that the studied semidiscrete method gives to one a good base for constructing and analyzing difference methods.

In Section 1 we give the statement of the initial-boundary value problems $\mathcal{P}_m$, $m = \overline{1,3}$, and formulate Theorem 1.1 on the existence of the generalized solutions. In Section 2, various estimates for solutions of auxiliary parabolic semidiscrete problem with boundary conditions of the third genus are given. After that, in Section 3, the semidiscrete method for solving problems $\mathcal{P}_m$ is constructed and Theorem 3.1 on global estimates on approximate solutions is formulated. Section 4 is devoted to its proof. In Section 5 we prove the convergence of approximate solutions and establish Theorem 1.1 by means of the passage to the limit.

Supported by the Russian Foundation for Basic Research (project codes 96-01-00621 and 97-01-00214).