ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 12, pp. 77–80. © Allerton Press, Inc., 2009.
Original Russian Text © A.A. Sedaev, 2009, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2009, No. 12, pp. 90–94.

Singular Symmetric Functionals and Stabilizing Subspaces of Marcinkiewicz Spaces

A. A. Sedaev[1*]
(Submitted by V. G. Zvyagin)

[1] *Voronezh State University of Architecture and Civil Engineering, ul. 20-Letiya Oktyabrya 84, Voronezh, 394006 Russia*

Received May 14, 2009

Abstract—We consider Marcinkiewicz spaces of functions measurable on a semiaxis that admit a wide set of singular symmetric Dixmier functionals. For elements of these spaces we study the measurability property introduced by A. Connes. We establish that this property is closely connected with the Tauberian property (which is more strong) but is not reduced to it. We specify the maximal subspace of the Marcinkiewicz space such that for its elements both properties are equivalent. We prove that this subspace is not reducible to other known subspaces of the Marcinkiewicz space and that it plays an important role in the theory of Dixmier functionals.

DOI: 10.3103/S1066369X09120123

Key words and phrases: *singular Dixmier traces, singular symmetric functionals, Connes measurable elements.*

The paper [1] by J. Dixmier has initiated the application of singular traces defined on ideals of compact operators in Hilbert spaces. Similarly to the ordinary trace, singular traces are invariant with respect to unitary similarity transformations, but they equal zero on all kernel operators, on which the ordinary operator trace exists and is finite. See the paper [2] by A. Connes for the main ideas and methods of the application of singular traces.

One of applications of singular traces is the description of the asymptotic behavior of s-numbers and eigenvalues of compact operators. Let S be some subset of singular traces. An element x is said to be S-measurable in the sense of Connes [2], if all functionals from S take on the same values on x.

From the outset, the study of elements S-measurable in the sense of Connes attracted a special attention [2–5]. This paper is a natural extension of [4] onto the alternating-sign case.

Although singular functionals have arisen and applied in noncommutative mathematics, they nevertheless have a nontrivial and rather important commutative analog [6–8]. In this case the singular traces are defined not on operators, but on elements of symmetric spaces of measurable functions [9]. It is customary to call such functionals *singular symmetric functionals*. This paper is written just in the framework of the commutative statement, but its results can be extended (in the standard way) onto symmetric ideals of measurable operators [6] associated with some semifinite von Neumann algebra. See the review [10] for examples of interrelations of commutative and noncommutative approaches.

Let ψ be a concave increasing continuous on the semiaxis function, $\psi(0) = 0$, $\psi(\infty) = \infty$, and let $M(\psi)$ be the corresponding Marcinkiewicz space, i.e., a Banach space of real measurable functions $x(s)$ with respect to the usual Lebesgue measure on the semiaxis $\mathbb{R}_+$, for which the Marcinkiewicz norm is finite

$$\|x\|_\psi = \sup_{0<t<\infty} \frac{1}{\psi(t)} \int_0^t x^*(s)ds;$$

here x^* denotes a nondecreasing permutation of the function $|x|$.

*E-mail: sed@vmail.ru.