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Intuitionistic Fuzzy Semiprime Ideals in Ordered Semigroups

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Abstract—We give characterizations of different classes of ordered semigroups by using intuitionistic fuzzy ideals. We prove that an ordered semigroup is regular if and only if every intuitionistic fuzzy left (respectively, right) ideal of S is idempotent. We also prove that an ordered semigroup S is intra-regular if and only if every intuitionistic fuzzy two-sided ideal of S is idempotent. We give further characterizations of regular and intra-regular ordered semigroups in terms of intuitionistic fuzzy left (respectively, right) ideals. In conclusion of this paper we prove that an ordered semigroup S is left weakly regular if and only if every intuitionistic fuzzy left ideal of S is idempotent.

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Key words and phrases: *intuitionistic fuzzy sets, intuitionitic fuzzy ideals, regular (respectively, intra-regular and left weakly regular) ordered semigroups.*

1. INTRODUCTION

The concept of a fuzzy set was introduced by L. A. Zadeh [1] in his classic paper published in 1965. Since then several researchers tried to generalize this fundamental concept. Unfortunately, these studies were not systematic. Fuzzy sets and intuitionistic fuzzy sets introduced by K. T. Atanassov [2] in his pioneer paper published in 1986 appeared to be highly useful to cope with vagueness. He also introduced [3] some new operations useful for studying the structure of intuitionistic fuzzy sets. For more details on intuitionistic fuzzy sets and their applications we refer to [4]. W. Gau and D. Buehrer [5] presented the concept of a vague set. However, P. Burillo and H. Bustince [6] have shown that the notions of a vague set and an intuitionistic fuzzy set are the same. Intuitionistic fuzzy sets also have many applications in various branches of mathematics. B. Davvaz et al. [7] applied this concept in the theory of H_v-modules. They introduced the notion of an intuitionistic fuzzy H_v-submodule of an H_v-module and studied its properties. K. Kim et al. [8] considered the intuitionistic fuzzification of the concept of sub-hyperquasigroups in a hyperquasigroup. In [9, 10] K. Kim and Y. Jun introduced the concept of intuitionistic fuzzy (interior) ideals of semigroups. Y. Jun [11] introduced the concept of intuitionistic fuzzy bi-ideals of ordered semigroups and characterized ordered semigroups in terms of this notion. M. Shabir and A. Khan [12] introduced the concept of intuitionistic fuzzy interior ideals of ordered semigroups and characterized the latter in these terms. In [13] we introduced the concept of intuitionistic fuzzy bi-ideals of subsets and intuitionistic fuzzy bi-filters and established some relations between prime intuitionistic fuzzy bi-ideals of subsets and intuitionistic fuzzy bi-filters of ordered semigroups.

Certain classes of ordered semigroups were studied in [14–16] in which, for example, regular ordered semigroups were characterized in terms of ideals.

In [17] one proposed to use fuzzy sets in ordered semigroups considering some basic characterizations of fuzzy prime ideals and fuzzy filters of ordered semigroups. On the other hand, fuzzy generalized bi-ideals and fuzzy ideals in semigroups are adduced in [18, 19].

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