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Smoothness of Sums of Trigonometric Series with Monotone Coefficients

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INTRODUCTION

In this paper we study the connection between the behavior of coefficients of trigonometric series with many variables and the smoothness of sums of these series in spaces L_p.

Let m be the space dimension, $T = [-\pi, \pi]$; assume that the function $f(\mathbf{x}) = f(x_1, \dots, x_m) \in L(T^m)$ is everywhere 2π-periodic in each variable. Let

$$\sum_{\mathbf{n}\in\mathbf{Z}^m} a_{\mathbf{n}}(f)e^{i\mathbf{nx}} = \sum_{\mathbf{n}\in\mathbf{Z}^m} a_{\mathbf{n}}e^{i\mathbf{nx}}$$

stand for the multiple Fourier series of the function $f(\mathbf{x})$, where $\mathbf{n} = (n_1, \dots, n_m)$, $\mathbf{x} = (x_1, \dots, x_m)$, $\mathbf{nx} = \sum\limits_{l=1}^{m} n_l x_l$.

Definition 1. Let a function $\omega(\delta)$ be nondecreasing and continuous on $[0,1]^m$, $\omega(\mathbf{0}) = 0$, $\omega(\delta+\nu) \le \omega(\delta) + \omega(\nu)$, for $0 \le |\delta| \le |\nu| \le |\delta| + |\nu| \le 1$. Then $\omega(\delta)$ is said to be the *module of continuity*.

The following theorem was proved by Hardy and Littlewood.

Theorem A. a) *If $1 < p \le 2$ and $f(\mathbf{x}) \in L_p(T^m)$, then*

$$\sum_{\mathbf{n}\in\mathbf{Z}^m} |a_{\mathbf{n}}(f)|^p \prod_{j=1}^{m} (|n_j|+1)^{p-2} \le c(p,m)\|f\|_p^p.$$

b) *If $2 \le p < \infty$ and the numbers $\{a_{\mathbf{n}}\}_{\mathbf{n}\in\mathbf{Z}^m}$ are such that*

$$J_p(a) = \Big(\sum_{\mathbf{n}\in\mathbf{Z}^m} |a_{\mathbf{n}}(f)|^p \prod_{j=1}^{m} (|n_j|+1)^{p-2}\Big)^{1/p} < \infty,$$

then a function $f(\mathbf{x}) \in L_p(T^m)$ exists such that for any $\mathbf{n} \in \mathbf{Z}^m$,

$$a_{\mathbf{n}}(f) = a_{\mathbf{n}} \ \ and \ \ \|f\|_p \le c(p,m)J_p(a).$$

For $m = 1$ this theorem is proved in [1], one can prove it for $m > 1$ by induction.

In addition, Hardy and Littlewood noted that in the one-dimensional case for the series $\sum\limits_{n=1}^{\infty} a_n e^{inx}$, where $a_1 \ge a_2 \ge \dots \ge 0$, $a_n \to 0$ as $n \to \infty$; a more precise assertion is also valid ([1], § 9.5).

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