# Кинетика установления термодинамического равновесия в ускоренной Вселенной при наличии скейлинга взаимодействий в унитарном пределе

Юрий Игнатьев

XV Российская гравитационная конференция
Казанский федеральный университет
Институт математики и механики им. Н.И. Лобачевского

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# History:

**First investigation in the given research area**

В 1986 году Автором было показано, что при условии скейлингового поведения сечений рассеяния элементарных частиц в области энергий выше унитарного предела в ультрарелятивисткой космологической плазме выживают в настоящую эпоху «истинно реликтовые частицы», образованные в момент рождения Вселенной с энергией выше

$$\mathrm{E} \gtrsim 10^{13}\mathrm{Gev}; \tag{1}$$

на современном этапе[a].

[a] Yu.G. Ignat'ev, Sov. Phys. J. (Izv. Vuzov Phys.), 1986, 29, p. 9-14.

**Further researches**

В 2006-2008 эти исследования были продолжены и на основе сформулированной модели энергобаланса были получены наблюдательные ограничения на параметры первичного неравновесного распределения[a], см. также [b]

[a] Yu.G. Ignat'ev, Gravitation & Cosmology, 13 (2007), p. 31-42;
Yu.G. Ignat'ev, D.Yu.Ignatyev, Gravitation & Cosmology, 13 (2007), p. 101-113;
Gravitation and Cosmology, 2008, 14, pp. 309-313.

[b] Yu.G.Ignatyev, R.A.Ziatdinov, Gravitation & Cosmology, Vol. 12 (2006), 293-302;
Yu.G.Ignat'ev, R.A.Ziatdinov, Gravitation and Cosmology, 2008, 14, pp. 301-308.

**Professor's Vitaly Melnikov Question**

В 2007 году на семинаре Gracos профессор Мельников задал Автору вопрос:
«... А какие поправки внесет в неравновесный сценарий учет современных наблюдательных данных об ускорении Вселенной?»
— Откровенно скажу, что тогда этот вопрос очень не понравился Автору. ... Однако, после некоторого размышления Автор решил ответить на этот вопрос...

# Cross section and Cinematics Invariantes

## Four-particles reactions kinematics $a + b \to c + d$

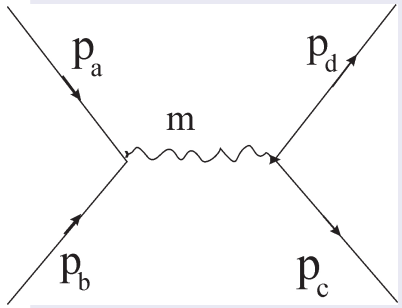


**Рис. 1.**

$$s = (p_a + p_b)^2 = (p_c + p_d)^2, \qquad (2)$$

$$t = (p_c - p_a)^2 = (p_b - p_d)^2 \qquad (3)$$

– кинематические инварианты (параметры Мандельстама). $\sqrt{s}$- энергия сталкивающихся частиц в центре масс (СЦМ), $F(s,t)$ – усредненные инвариантные амплитуды рассеяния.

## Total cross-section of reaction

$$\sigma_{tot} = \frac{1}{16\pi\lambda^2(s, m_a^2, m_b^2)} \int_{t_{min}}^{0} dt |F(s,t)|^2, \qquad (4)$$

$|F(s,t)|^2$ – квадрат усредненной по всем состояним амплитуды рассеяния, $\lambda$ - функция треугольника $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$; $t_{min} = -\frac{\lambda^2}{s}$.

## Ultrarelativistic limit $s/m^2 \to \infty; \quad \lambda \to s^2; \; x = -t/s$

$$\sigma_{tot}(s) = \frac{1}{16\pi s} \int_{0}^{1} dx |F(s,x)|^2. \qquad (5)$$

*Полное сечение рассеяния зависит лишь от кинематического инварианта $s$, зависимость которого от времени и определяет временную зависимость полного сечения рассеяния:*

$$\sigma_{tot} = \sigma_{tot}(s). \qquad (6)$$

# General assumptions of the model:

**$1^o$. An asymptotic scaling character of elementary particles interaction in range of extra-high energies:**

$1^o$a. A restoration of elementary particles' *scaling of interactions* at energies above the *unitary limit*:

$$\sigma|_{s>s_u} \sim \frac{1}{s}; \tag{7}$$

$1^o$b. *unification of all interparticle interactions* on basis of fundamental constants $G, \hbar, c$ at energies above the unitary limit:

$$\sigma_0(s) = \frac{8\pi}{s\Lambda(s)},\ \Lambda(s) = \ln^2\left(1+\frac{s_0}{s}\right) \Rightarrow 1+\ln^2\left(1+\frac{s_0}{s}\right), \tag{8}$$

$s_0 = 4$ is a squared total energy of two colliding Planck masses in the ordinary units, factor $\beta \sim 1$.

**$2^o$. *Minimality of fundamental fields' coupling with a cosmological plasma*:**

$$\dot{\varepsilon_s} + 3\frac{\dot{a}}{a}(\varepsilon_s + \mathrm{p}_s(\varepsilon_s)) = 0,\ \dot{\varepsilon_p} + 3\frac{\dot{a}}{a}(\varepsilon_p + \mathrm{p}_p(\varepsilon_p)) = 0. \tag{9}$$

**$3^o$. *Ultrarelativistic state equation of cosmological plasma* at the stage of expansion that is being researched:**

$$\mathrm{p}_p = \frac{1}{3}\varepsilon_p, \quad (9) \Rightarrow \varepsilon_p a^4 = \mathrm{Const}. \tag{10}$$

**$4^o$. *Ultrarelativistic start of the Universe*:**

$$\lim_{t\to 0} \varkappa = \frac{1}{3}, \qquad \varkappa \equiv p(\varepsilon)/\varepsilon. \tag{11}$$

# Asymptotic Cross Section (ACS)

**Comparison of the universal cross-section of scattering (8) at factor $\beta = 1$ to the well-known cross-sections of fundamental processes (Fig. 2) – bold line**

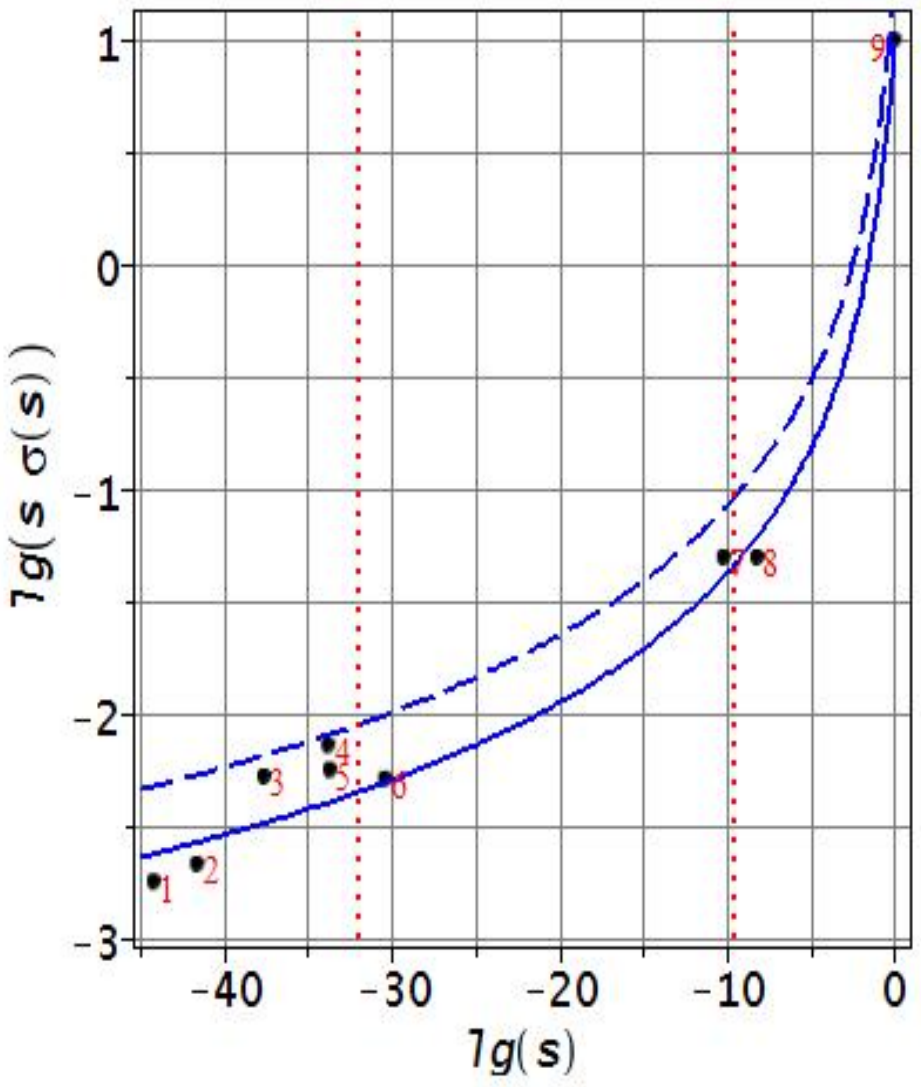


Fig. 2. Dotted line corresponds to the graph of universal cross-section of scattering at factor $\beta = 2$. On the abscissa axis are laid values of the common logarithm of the first kinematic invariant $\lg s$ in Planck units; along the ordinate axis are laid values of the common logarithm of the dimensionless invariant, $\lg s\sigma(s)$. 1 – Thompson scattering, 2 – Compton scattering on electrons at $E_{SCM} = 10$ Mev, 3 – Compton scattering of electrons at $E_{SCM} = 1$ Gev, 4 – electroweak interaction with participation of: $W$ - bosons, 5 – with participation of $Z$ - bosons, 6 – with participation of H-bosons at energy of the order of 7 Tev ($\sigma \sim 10$fb); 7 – $SU(5)$ - interaction at mass of the superheavy X-bosons $10^{15}$ Gev, 8 – $10^{14}$ Gev; 9 – gravitational interaction on Planck scales. Vertical dotte lines correspond to energy values of the unitary limit for $SU(2) \times SU(1)$ - interactions, $E_u = 617$ Gev, and SU(5) - interactions, $E_u \sim 10^{14}$ Gev.

# Energy balance of the cosmological plasma

$$(8) \Rightarrow (10) \Rightarrow \varepsilon_p a^4 \equiv \tilde{\varepsilon}_p = \text{Const}, \tag{12}$$

$\tilde{\varepsilon}_p$ is a conformal energy density of the cosmological plasma;

$$\varepsilon_p = \varepsilon_e + \varepsilon_{ne}, \tag{13}$$

$\varepsilon_e$ is a energy density of the equilibrium components of plasma, and $\varepsilon_{ne}$ a energy density of the nonequilibrium components of ultrarelativistic plasma:

$$\mathrm{p}_e = \frac{1}{3}\varepsilon_e; \quad \mathrm{p}_{ne} = \frac{1}{3}\varepsilon_{ne}. \tag{14}$$

Let us determine this constant, setting according to (11) on the initial ultrarelativistic stage of expansion:

$$a(t)|_{t\to 0} = \sqrt{t}, \Rightarrow \tilde{\varepsilon}_p = \frac{3}{32\pi}. \tag{15}$$

Let us then introduce temperature $T_0(t)$ of the cosmological plasma in the ideal Universe, in which at this point in cosmological time $t$ the whole plasma is locally balanced. Thus, plasma's energy density is equal to

$$\varepsilon_p^0 = \frac{N_0 \pi^2}{15} T_0^4(t), \tag{16}$$

where $N_0 = N_B^0 + 1/2 N_F^0$ is an effective number of equilibrium particles' types (bosons and fermions) in plasma with temperature $T_0$.

Hence subject to (15) we obtain the evolution law of plasma's temperature in the equilibrium Universe:

$$T_0(t) = \frac{1}{a(t)} \left( \frac{45}{32\pi^3 N_0} \right)^{\frac{1}{4}}. \tag{17}$$

# Energy balance of the cosmological plasma

With respect to $N_0$ — *effective number of types* of thermodynamically equilibrium particles, we suggest that $N_0(t)$ is a slowly changing function of the cosmological time:

$$\dot{N}_0 t \ll 1. \tag{18}$$

Let then $T(t)$ is a real temperature of equilibrium component of cosmological plasma, and $\Delta f_a(p,t)$ is a distribution function of «a»-sort nonequilibrium plasma particles. Then we find energy densities of the equilibrium, $\varepsilon_e$, and nonequilibrium, $\varepsilon_{ne}$, components:

$$\varepsilon_e = \frac{N\pi^2}{15} T^4(t); \tag{19}$$

$$\varepsilon_{ne} = \frac{1}{2\pi^2} \sum_a (2S+1) \int_0^\infty p^3 \Delta f_a(p,t) dp, \Rightarrow \tag{20}$$

$$\tilde{\varepsilon}_{ne} = \frac{45}{64\pi^5} \sum_a (2S+1) \int_0^\infty \tilde{p}^3 \Delta f_a(\tilde{p},t) d\tilde{p}, \tag{21}$$

where $N(t) = N_B + 1/2 N_F$ is an effective number of equilibrium particles' (bosons and fermions) types in plasma with temperature $T(t)$ and introduced new *dimensionless* conformal momentum variable $\tilde{p}$:

$$p = \left(\frac{45}{32\pi^3}\right)^{\frac{1}{4}} \cdot \frac{\tilde{p}}{a(t)} = T_0(t) N_0^{\frac{1}{4}} \tilde{p}. \tag{22}$$

Next, from (17) and (19) we obtain for the conformal energy density of the equilibrium component of plasma:

$$\tilde{\varepsilon}_e = \frac{3}{32\pi} y^4, \quad y(t) = \frac{T(t)}{T_0(t)} \leqslant 1, \tag{23}$$

# Energy balance of the cosmological plasma

From (23) it is possible to obtain the following relation:

$$\sigma(t) \equiv y^4(t) \equiv \frac{\varepsilon_e}{\varepsilon_p} \equiv \frac{\tilde{\varepsilon}_e}{\tilde{\varepsilon}_e + \tilde{\varepsilon}_{ne}}. \tag{24}$$

Thus, cosmological plasma's energy conservation law (12) with a use of (20) and (23) can be rewritten in form:

$$y^4 + \frac{15}{2\pi^4} \sum_a (2S+1) \int_0^\infty \tilde{p}^3 \Delta f_a(\tilde{p}, t) d\tilde{p} = 1. \tag{25}$$

Relation (25) is called *plasma energy-balance equation*. It is obtained with a usage of three model suggestions — $2^o$, $3^o$, $5^o$. Let us note that in previous articles this basic relation of mathematical model of thermodynamical equilibrium's restoration has been obtained under more special suggestions. At given dependency of nonequilibrium particles' distribution function on the temperature of plasma's equilibrium component and cosmological time the energy-balance equation becomes a nonlinear integral equation relative to equilibrium component's temperature. Therefore, to obtain this equation in the explicit form it is necessary to solve the kinetic equation for the nonequilibrium particles.

# Solution of kinetic equation

As was shown in (Yu.G. Ignatyev, 2007), kinetic equation for superthermal particles in ultrarelativistic plasma has a form:

$$p\frac{\partial \Delta f_a}{\partial t} = -\frac{4\pi N}{3}\frac{T^2(t)}{\Lambda(pT/2)}\Delta f_a, \tag{26}$$

where $\Delta f_a = \Delta f_a(\tilde{p}, t)$, $\Lambda = \Lambda(s)$. Using here relation (22), we reduce (26) to form:

$$\frac{\partial \Delta f_a}{\partial t} = -\frac{8\pi N}{3\tilde{p}\Lambda(\frac{1}{2}\tilde{p}\ T_0 T N^{1/4})}\left(\frac{2\pi^3}{45}\right)^{1/4} T^2(t)a(t)\Delta f_a. \tag{27}$$

Solving (27), obtain:

$$\Delta f_a(t,\tilde{p}) = \Delta f_a^0(\tilde{p})\exp\left[-\frac{8\pi}{3\tilde{p}}\left(\frac{2\pi^3}{45}\right)^{1/4}\int\limits_0^t \frac{NaT^2dt}{\Lambda(\frac{1}{2}\tilde{p}\ T_0 T N_0^{1/4})}\right], \tag{28}$$

where

$$\Delta f_a^0(\tilde{p}) \equiv \Delta f_a(0,\tilde{p}). \tag{29}$$

# Solution of kinetic equation

**Transition to dimensionless normalized variables**

Let us introduce *an average conformal energy of ultrarelativistic particles' nonequilibrium component at the initial point in time*, $\langle\tilde{p}\rangle_0$, –

$$\langle\tilde{p}\rangle_0 = \frac{\tilde{\varepsilon}(0)}{\tilde{n}(0)} \equiv \frac{\sum\limits_a (2S+1)\int\limits_0^\infty \Delta f_a^0(\tilde{p})\tilde{p}^3 d\tilde{p}}{\sum\limits_a (2S+1)\int\limits_0^\infty \Delta f_a^0(\tilde{p})\tilde{p}^2 d\tilde{p}} \tag{30}$$

and *a dimensionless normalized momentum variable*, $\rho$, –

$$\rho \equiv \frac{\tilde{p}}{\langle\tilde{p}\rangle_0}, \tag{31}$$

so that

$$\langle\rho\rangle_0 = \frac{\tilde{\varepsilon}(0)}{\langle\tilde{p}\rangle_0\tilde{n}(0)} \equiv 1 \Rightarrow \langle\rho\rangle_0 = \frac{\sum\limits_a (2S+1)\int\limits_0^\infty \Delta f_a^0(\rho)\rho^3 d\rho}{\sum\limits_a (2S+1)\int\limits_0^\infty \Delta f_a^0(\rho)\rho^2 d\rho} = 1. \tag{32}$$

# Transition to dimensionless normalized variables

$$\langle \tilde{p} \rangle_0 \gg 1. \tag{33}$$

$$\Lambda\left(\frac{1}{2}\tilde{p}\; T_0 T N_0^{1/4}\right) \simeq \Lambda(\langle \tilde{p} \rangle_0 T_0^2) \equiv \Lambda_0(t). \tag{34}$$

Thus, with a logarithmic accuracy we represent a solution (28) in compact form:

$$\Delta f_a(t,\rho) = \Delta f_a^0(\rho) \exp\left(-\frac{2}{\rho}\int\limits_0^t \xi \frac{y^2}{a} dt\right), \tag{35}$$

where it is introduced the denotation:

$$\xi \equiv \xi(t) = \left(\frac{5\pi}{18}\right)^{1/4} \frac{N}{\langle \tilde{p} \rangle_0 N_0^{1/2} \Lambda_0(t)}$$

$$\approx 0.967 \frac{N}{\langle \tilde{p} \rangle_0 N_0^{1/2} \Lambda_0(t)} \approx \frac{N}{\langle \tilde{p} \rangle_0 N_0^{1/2} \Lambda_0(t)}. \tag{36}$$

# Transition to dimensionless normalized variables

Introducing then a new *dimensionless time variable*, $\tau$, –

$$\tau = 2 \int_0^t \frac{\xi}{a}\, dt\ , \tag{37}$$

so that:

$$\frac{d\tau}{dt} \equiv 2\frac{\xi}{a} > 0, \tag{38}$$

and a new *dimensionless function*, $Z(\tau)$, –

$$Z(\tau) = \int_0^\tau y^2(\tau) d\tau, \tag{39}$$

we reduce the kinetic equation's (35) solution to form:

$$\Delta f_a(\tau, \rho) = \Delta f_a^0(\rho) \cdot \mathrm{e}^{-\frac{Z(\tau)}{\rho}}\ . \tag{40}$$

# Transition to dimensionless normalized variables

## Conformal energy density of the nonequilibrium component

$$\tilde{\varepsilon}_{ne} = \frac{45}{64\pi^5}\sum_a (2S+1)\int_0^\infty \tilde{p}^3 \Delta f_a^0(\rho) \mathrm{e}^{-\frac{Z(\tau)}{\rho}}. \tag{41}$$

Let us perform an identity substitution on the given expression, accounting that subject to the definition (24)and energy-balance equation (25):

$$\tilde{\varepsilon}^0_{ne} = (1-\sigma_0)\frac{3}{32\pi}: \tag{42}$$

$$\tilde{\varepsilon}_{ne} \equiv \frac{\tilde{\varepsilon}_{ne}}{\tilde{\varepsilon}^0_{ne}}\tilde{\varepsilon}^0_{ne} = (1-\sigma_0)\Phi(Z)\frac{3}{32\pi}, \tag{43}$$

where we introduced new *dimensionless function* $\Phi(Z)$ with a use of transformation to the dimensionless momentum variable $\rho$ (31) :

$$\Phi(Z) \equiv \frac{\sum_a (2S+1)\int_0^\infty \rho^3 \Delta f_a^0(\rho) \mathrm{e}^{-\frac{Z(\tau)}{\rho}}}{\sum_a (2S+1)\int_0^\infty \rho^3 \Delta f_a^0(\rho)}. \tag{44}$$

# Energy-balance equation solution and analysis

As a result of definition (39) function $Z(\tau)$ satisfies following conditions:

$$Z'(\tau) = y^2(\tau) \Rightarrow Z'^{\,2} = \sigma(\tau); \tag{45}$$

$$Z(0) = 0; \quad Z'(0) = y^2(0) = \sqrt{\sigma_0}, \tag{46}$$

where

$$Z' \equiv \frac{dZ}{d\tau} > 0. \tag{47}$$

Thus, subject to (43) – (45) energy-balance equation (25) can be rewritten in form of the differential equation relative to function $Z(\tau)$:

$$\begin{aligned} y^4 + (1-\sigma_0)\Phi(Z) &= 1 \Rightarrow \\ Z'^2 + (1-\sigma_0)\Phi(Z) &= 1, \end{aligned} \tag{48}$$

# Energy-balance equation solution and analysis

Solving it subject to relations (46) – (47), we find a formal solution in the implicit form:

$$\int_0^Z \frac{du}{\sqrt{1-(1-\sigma_0)\Phi(u)}} = \tau. \qquad (49)$$

$$\Phi(Z) > 0, (Z \in [0, +\infty)), \Phi(0) = 1; \quad \lim_{Z \to +\infty} \Phi(Z) = 0. \qquad (50)$$

$$\Phi(Z)'_Z < 0, (Z \in [0, +\infty)); \Phi(Z)'' > 0, (Z \in [0, +\infty)). \qquad (51)$$

Function $\Phi(Z)$ is strictly monotonously decreasing and is limited on the interval:

$$\Phi(Z) \in [0, 1]; \quad (Z \in [0, +\infty)), \qquad (52)$$

and function $\Phi(Z)$ has a concave graph. Always has a single and just only one solution $Z = Z_0$, i.e. *mapping* $Y = \Phi(Z)$ *on the set of nonnegative numbers is bijective*.

# Energy-balance equation solution and analysis

Differentiating relation (48) by $\tau$ like a composite function, we find:

$$Z'[2Z'' + (1-\sigma_0)\Phi'_Z] = 0. \tag{53}$$

Hence as a result of $Z'$ positivity (48) let us find the second derivative:

$$Z'' = -\frac{1}{2}(1-\sigma_0)\Phi'_Z. \tag{54}$$

Therefore in consequence of (51) and(**??**) – (24) we obtain from (54):

$$Z'' > 0, \tag{55}$$

# Energy-balance equation solution and analysis

i.e. graph of $Z(\tau)$ function is also concave. Then, differentiating (IV.53), subject to (55) find:

$$y' > 0, \tag{56}$$

— i.e. function $y(\tau)$ (and function $\sigma(\tau)$ together with it) is a monotonously increasing one. From the other hand it is limited from below by the initial value $y_0$ ($\sigma_0$), and from above by value $1$:

$$y' > 0, y \in [y_0, 1); \quad \sigma' > 0, \sigma \in [\sigma_0, 1). \tag{57}$$

Listed properties of functions $y(\tau)$, $Z(\tau)$ and $\Phi(Z)$ assert the bijectivity of chain of mappings $\tau \leftrightarrow y$, $y \leftrightarrow Z$, $Z \leftrightarrow \Phi$. To close this chain it is enough to determine functions $y(\tau)$ and $Z(\tau)$ coupling by means of the energy-balance equation(49):

$$y = [1 - (1 - \sigma_0)\Phi(Z)]^{1/4}. \tag{58}$$

# Energy-balance equation solution and analysis

Equations (49) and (58) are the *parametric solution of the energy-balance equation* (48), and above mentioned properties of functions $\Phi(Z)$ and $Z(\tau)$ assert the *uniqueness of the solution*. According to (44) function $\Phi(Z)$ is fully determined by the initial distribution of nonequilibrium particles $\Delta f_a^0(\rho)$. Therefore from the mathematical point of view the problem of thermodynamical equilibrium recreation in Universe with arbitrary acceleration is completely solved. Concrete models are determined by dark matter model and model of initial nonequilibrium distribution of particles.

Let us differentiate now the relation (54) by $\tau$ and take account of the association (45) between $y(\tau)$ and $Z(\tau)$:

$$Z''' = -\frac{1}{2}(1-\sigma_0)\Phi''_{ZZ}Z'$$

$$\Rightarrow y''y = -y'^2 - \frac{1}{4}(1-\sigma_0)\Phi''_{ZZ}y^2.$$

# Energy-balance equation solution and analysis

Thus, as a result of (51):

$$y'' < 0, \tag{59}$$

— i.e. graph of function $y(\tau)$, and $\sigma(\tau)$ graph together with it, are concave. Then since $\Phi_Z(Z \to \infty) = 0$, from (54) it follows:

$$\lim_{\tau \to \infty} y'(\tau) = 0 \Rightarrow \lim_{\tau \to \infty} \sigma'(\tau) = 0, \tag{60}$$

— i.e., value $\sigma = 1$ is reached asymptotically at $\tau \to \infty$. This allows to draw a qualitative graph of functions $y(\tau)$ (Fig. 3). Finiteness of the dimensionless time $\tau_\infty$ conducts to the establishment of the limiting value of function $y(\tau)$:

$$y(\tau_\infty) = y_\infty < 1 \Rightarrow \lim_{t \to \infty} y(t) = y_\infty < 1. \tag{61}$$

Details and numeric modelling see[1]

[1] *Yu. G. Ignatyev, Gravitation and Cosmology, 2013, 19, pp. 232-239. DOI: 10.1134/S0202289313040087; Yu. G. Ignat'ev, Russian Physics Journal, 56, 2013. - p. 693-706; Yu. G. Ignatyev, Gravitation and Cosmology, 2014, 20, pp. 99-105. DOI: 10.1134/S0202289314020066.*

# Energy-balance equation solution and analysis

In consequence of that the certain part of comsological plasma energy is forever conserved in the nonequilibrium superthermal component:

$$\lim_{t\to\infty}\frac{\varepsilon_{ne}(t)}{\varepsilon_p(t)} = 1-\sigma_\infty = \begin{cases} =0, & \tau_\infty=\infty \\ >0, & \tau_\infty<\infty \end{cases} . \qquad (62)$$

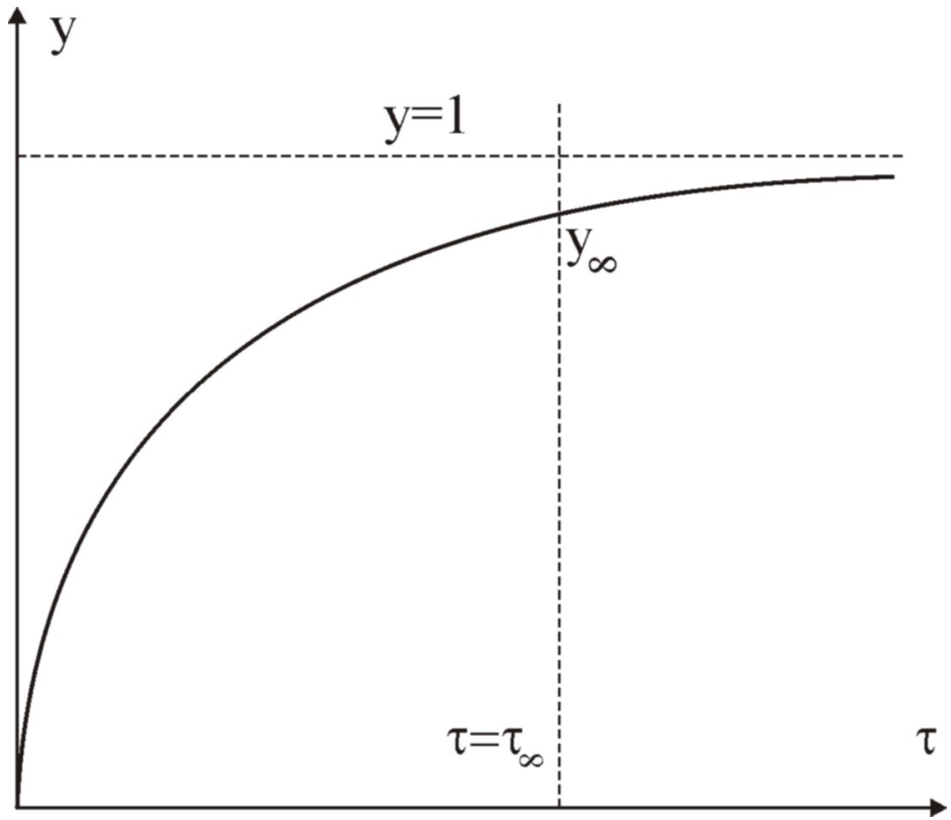


Рис.: Qualitative form of function's $y(\tau)$ graph.

## Exact model of transition from the ultrarelativistic stage to the inflationary one

Let us consider a simple model of matter consisting of 2 components – minimally coupled massive scalar field (cosmological member) with a state equation:

$$p_s = -\varepsilon_s, \tag{63}$$

and the ultrarelativistic plasma with the state equation (10). Then summary barotropic factor and invariant acceleration can be written in form: Energy conservation laws (9) – (8) take form:

$$\varepsilon_s = \mathrm{Const} = \frac{3\Lambda^2}{8\pi}; \tag{64}$$

$$\varepsilon_p a^4 \equiv \tilde{\varepsilon}_p = \mathrm{Const} \simeq \frac{3}{32\pi}. \tag{65}$$

Substituting (64)-(65 )into the Einstein equation and integrating it, we obtain:

$$a(t) = \frac{1}{\Lambda}\sqrt{\frac{3}{32\pi}\mathrm{sh}\frac{t}{2\Lambda}} \tag{66}$$

## Exact model of transition from the ultrarelativistic stage to the inflationary one

Next, according to(63) it is possible to calculate an effective barotropic factor and an invariant acceleration (see Figure 4).

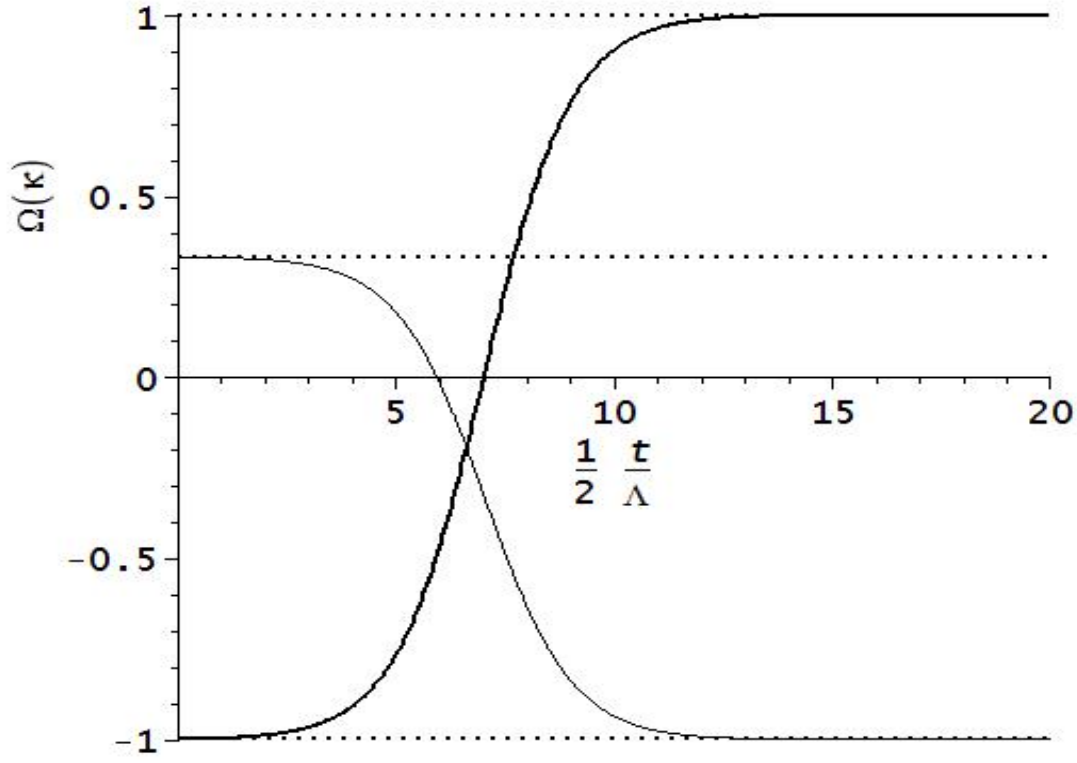


Рис.: Evolution of the effective barotropic factor $\kappa(t)$ (thin line) and invariant acceleration $\Omega(t)$ (heavy line) relative to the exact solution (66) at $\Lambda = 1$. Asymptotes $-1;\ 1/3;\ 1$ are denoted by the dotted lines.

In order to construct a numerical model let us consider the initial distribution of white noise type:

$$\Delta f_0(\rho) = \frac{A}{\rho^3}\chi(\rho_0 - \rho), \tag{67}$$

where $A$ is a normalization constan, $\rho_0 > 1$ is a dimensionless parameter, $\chi(x)$ is a Heaviside step function, so that the conformal energy density with respect to this distribution is equal to:

$$\tilde{\varepsilon}^0_{ne} = \frac{\langle\tilde{p}\rangle^4_0 A\rho_0}{32\pi^5}. \tag{68}$$

Calculating function $\Phi(Z)$ relative to distribution (67), we find:

$$\Phi(Z) = e^{-x} - x\mathrm{Ei}(x); \quad x \equiv \frac{Z}{\rho_0}, \tag{69}$$

where $\mathrm{Ei}(x)$ is an integral exponential function

$$\mathrm{Ei}(x) = \int\limits_{-1}^{\infty} \frac{e^{-tx}}{t} dt. \tag{70}$$

## The results of numerical integration

In particular, the integration of the relation (37) confirmed insensitivity of value $\tau_\infty$ from the number of parameters and, practically, confirmed the estimation formula (37)

$$\tau_\infty = \lim_{t\to+\infty} \tau(t) = \frac{2\Lambda\langle\xi\rangle}{\langle\tilde{p}\rangle_0}\mathrm{F}(1, 1/\sqrt{2}), \tag{71}$$

which did not account the details of the logarithmical dependency of the parameter $\langle\xi\rangle$ on time. According to (40)

$$\Delta f_a(\tau, \rho) = \Delta f_a^0(\rho) \cdot \mathrm{e}^{-\frac{Z(\tau)}{\rho}}, \tag{72}$$

this means that at $t \to \infty$ superthermal particles' distribution is "frozen":

$$\Delta f_a(\tau_\infty, \rho) = \Delta f_a^0(\rho) \cdot \mathrm{e}^{-\frac{Z_\infty}{\rho}}. \tag{73}$$

# The results of numerical integration

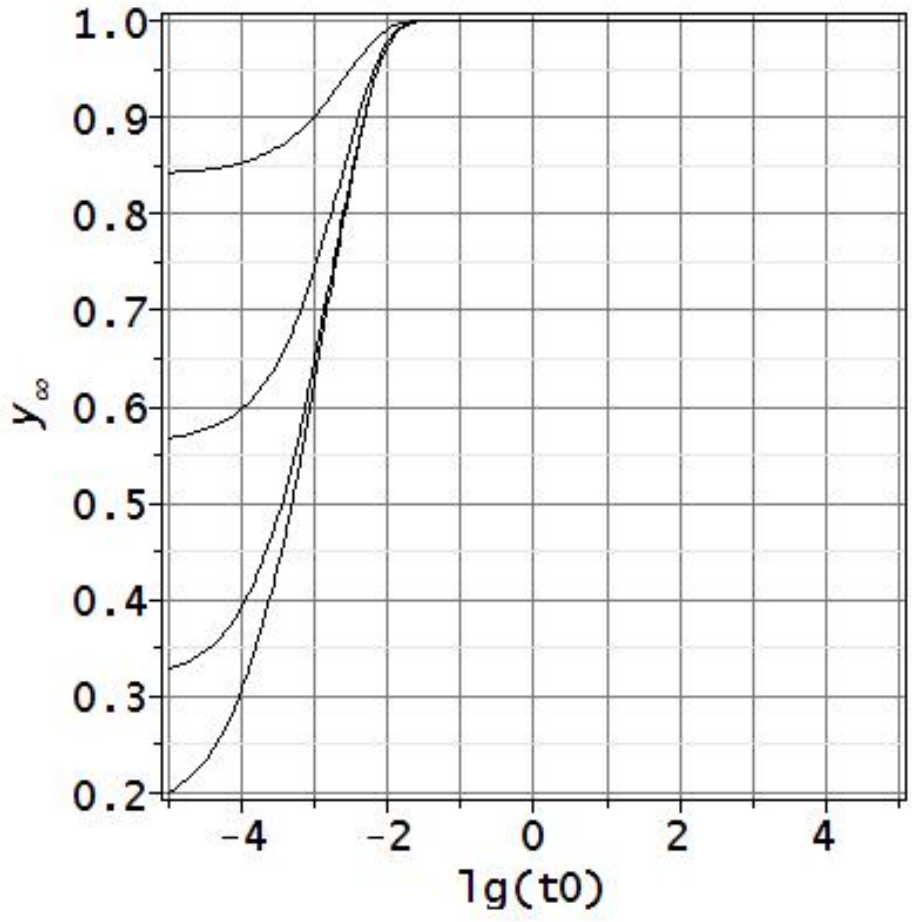


Рис.: 4. Dependency of the relative temperature $y_\infty$ on cosmological constant $t_0 = 4\Lambda$; it is everywhere $t_0 = 1$; $N_0 = 100$; $N = 10$; $\sigma_0$ — bottom-up: 0.001; 0.01; 0.1; 0.5.

# The results of numerical integration

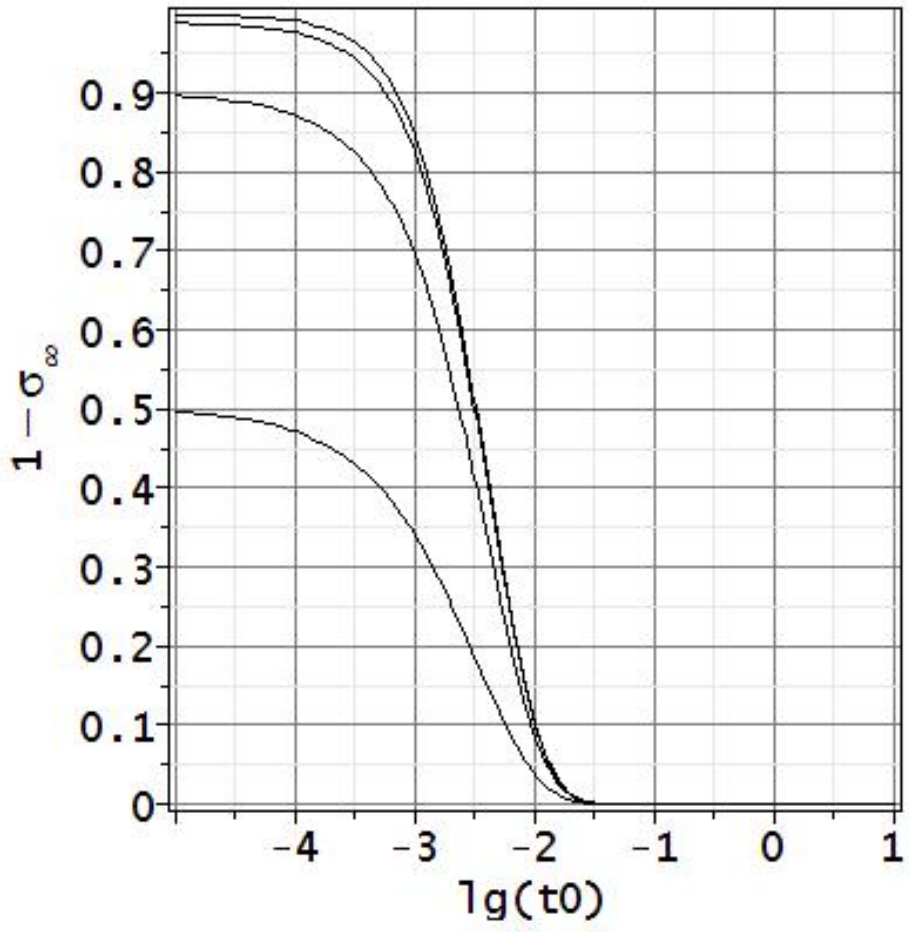


Рис.: Dependency of a relative part of energy of nonequilibrium particles, $1 - \sigma_\infty$, on cosmological constant $t_0$; it is everywhere $\langle \tilde{p} \rangle_0 = 1000$; $N_0 = 100$; $N = 10$, $\sigma_0$ — top-down: 0.001; 0.01; 0.1; 0.5.

# The results of numerical integration

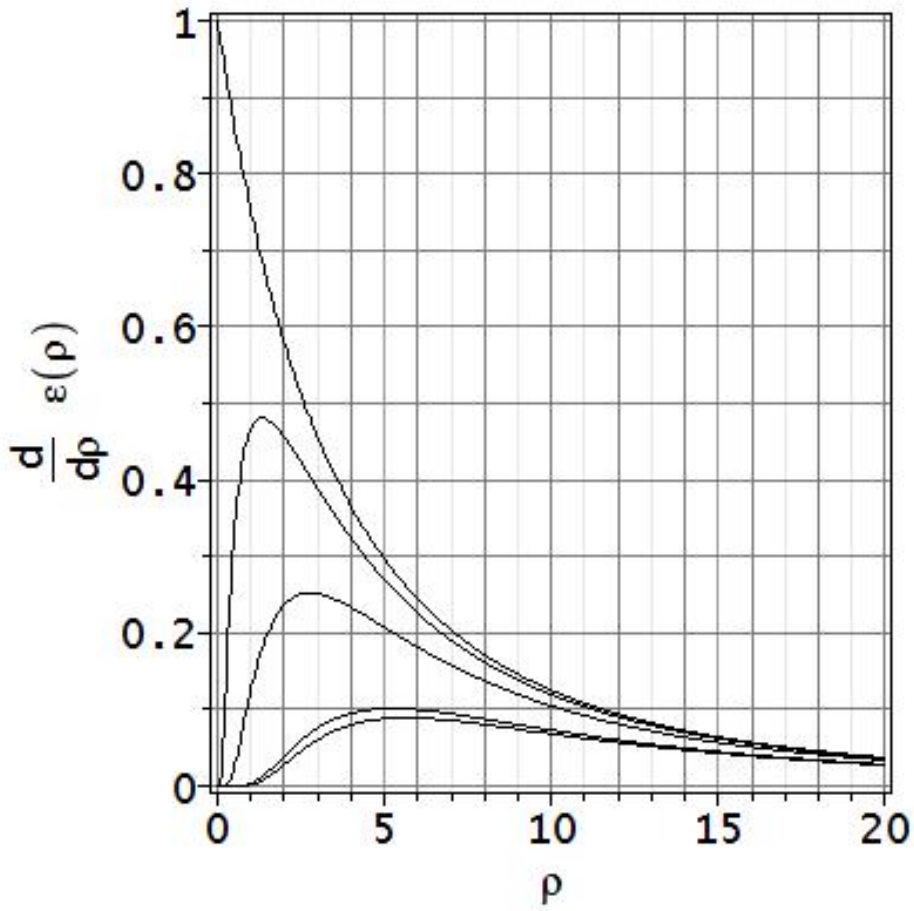


Рис.: 5. Evolution of density of distribution of energy; it is everywhere $\sigma_0 = 0.5$; $N_0 = 100$; $N = 10$, $\langle\tilde{p}\rangle_0 = 10$, $t_0 = 1$; from top to down: t= 0.0001; 0.01; 0.1; 1; 10; 100. Lines at t=10 and t=100 are coincide.

# Basic Statements of Relativistic Kinetic Theory

Центральным объектом релятивистской кинетической теории является *инвариантная функция распределения* $f_a(x,p)$ «a»-го сорта частиц, определяющая соответствующие

**Macroscopic moments of Distribution Function:**

$$n_a^i = \int\limits_{P_a(x)} f_a(x,p)p^i dP_a; \tag{74}$$

$$T_a^{ik} = \int\limits_{P_a(x)} f_a(x,p)p^i p^k dP_a, \tag{75}$$

$n^i$ – плотность потока числа частиц, $T^{ik}$ – тензор энергии - импульса частиц сорта «a»; $dP_a$ – инвариантный нормированный элемент объема пространства импульсов:[2]

$$dP = \sqrt{-g}\frac{(2S+1)}{(2\pi)^3}\frac{dp^1 dp^2 dp^3}{p_4}, \tag{76}$$

$(2S+1)$ – фактор вырождения ($S$ – спин), $p_4$ – положительный ковариантный корень уравнения массовой поверхности $(p,p) = m^2$. Пусть в плазме протекают реакции:

$$\sum_{A=1}^{m} \nu_A a_A \rightleftarrows \sum_{B=1}^{m'} \nu'_B a'_B\ , \tag{77}$$

где $\nu_A$ - числа частиц сорта $a_A$, участвующих в реакции; $A$ - номер частицы.

[2] *Всюду выбрана система единиц:* $\hbar = c = G = 1$.

# Kinetic Equations

**Distribution Functions are determined by the relativistic Kinetics Equations:**

$$\frac{df_a}{ds} \equiv \tilde{\nabla}_i f_a \frac{\partial \mathcal{H}_a}{\partial p_i} - \tilde{\nabla}_i \mathcal{H}_a \frac{\partial f_a}{\partial p_i} = I_a(x, p_a)\,, \tag{78}$$

где $\mathcal{H}_a(x,p)$ - функция Гамильтона, $\tilde{\nabla}$ – ковариантная производная в фазовом расслоении (производная Картана), $I_a(x,p_a)$ – релятивистский интеграл столкновений:

$$I_a(x p_a) = -\sum_a \nu_a \int \delta^{(4)}\left(p_f - p_i\right)\left(Z_{if} W_{if} - Z_{fi} W_{fi}\right) \prod_{i,f}{}' dP\,, \tag{79}$$

– $Z_{if}, Z_{fi}$ – статистические факторы начальных и конечных состояний реакций:

$$Z_{if} = \prod_{A=1}^{m} \prod_{\alpha=1}^{\nu_A} f_A(x, p_A^{\alpha}) \prod_{B=1}^{m'} \prod_{\beta=1}^{\nu'_B} \left[1 \pm f_B(x, p_B'^{\beta})\right];$$
$$Z_{fi} = \prod_{A=1}^{m} \prod_{\alpha=1}^{\nu_A} \left[1 \pm f_A(x, p_A^{\alpha})\right] \prod_{B=1}^{m'} \prod_{\beta=1}^{\nu'_B} f_B(x, p_B'^{\beta})\,,$$

– $\prod'_{i,f} dP$ – произведение импульсных объемов всех частиц кроме «$a$» – той; $W_{if}, W_{fi}$ – фазовые плотности вероятности перехода; $M_{if}$ инвариантные нормированные амплитуды рассеяния:

$$W_{if} = (2\pi)^4 |M_{if}|^2 2^{-(\sum \nu_A + \sum \nu'_B)}\,, \tag{80}$$

# Conservation Laws

**Kinetic equations' strict consequences are conservation laws of moments (74,75) :**

$$\nabla_i J^i \equiv \nabla_i \sum_a q_a n_a^i = 0; \tag{81}$$

$$\nabla_k T^{ik} \equiv \nabla_k \sum_a T_a^{ik} = 0, \tag{82}$$

где $J^i$ – вектор плотности тока, $q_a$ – сохраняющиеся фундаментальные заряды; $T^{ik}$ – суммарный тензор энергии-импульса плазмы.

**При выполнении условия ЛТР**

$$\tau_{eff} \ll t, \tag{83}$$

где $\tau_{eff}$ – эффективное время между столкновениями частиц,$t$ – характерное время эволюции системы, в плазме устанавливается ЛТР. При этом функции распределения частиц принимают локально-равновесный вид:

$$f_a^0(x, p_a) = \{\exp[-\lambda_a + (\xi, p_a)] \mp 1\}^{-1}\ , \tag{84}$$

где верхний знак соответствует бозонам, нижний - фермионам;

$$(\xi, \xi) > 0;\ \theta = 1/\sqrt{(\xi, \xi)};\quad v^i = \xi^i/\sqrt{(\xi, \xi)},\quad \mu_a = \lambda_a \theta; \tag{85}$$

– $\theta$ – локальная температура, $v^i$ – макроскопическая скорость, $\mu_a$ – химический потенциал.

# Local Thermodynamic Equilibrium (LTE)

В условиях ЛТР химические потенциалы для реакций (77) должны удовлетворять серии условий химического равновесия:

$$\sum_{A=1}^{m} \nu_A \mu_A = \sum_{B=1}^{m'} \nu'_B \mu'_B \,. \tag{86}$$

**Macroscopic moments with a relation to the equilibrium distribution function ( $f_0$ ) are equal to:**

$$n_a^i = n_a v^i; \tag{87}$$

$$T^{ik} = (\varepsilon + p) v^i v^k - p g^{ik}, \tag{88}$$

где $n_a$ – плотность числа частиц сорта «a» в сопутствующей системе отсчета, $\varepsilon$ и $p$ – суммарные плотность энергии и давление плазмы.

Таким образом, в состоянии ЛТР тензор энергии-импульса плазмы имеет гидродинамическую структуру идеальной жидкости (88), т.е., в сопутствующей системе отсчета *изотропен*. Равновесные *скалярные* плотности $n_a$, $\varepsilon_a$ и $p_a$ равны:

$$n_a(x) = \frac{2S+1}{2\pi^2} \int_0^\infty \left\{ \exp\left[ \frac{-\mu_a + \sqrt{m^2+p^2}}{\theta} \right] \mp 1 \right\}^{-1} p^2 dp \,;$$

$$\varepsilon_a(x) = \frac{2S+1}{2\pi^2} \int_0^\infty \left\{ \exp\left[ \frac{-\mu_a + \sqrt{m^2+p^2}}{\theta} \right] \mp 1 \right\}^{-1} \sqrt{m^2+p^2} p^2 dp \,;$$

$$p_a(x) = \frac{2S+1}{6\pi^2} \int_0^\infty \left\{ \exp\left[ \frac{-\mu_a + \sqrt{m^2+p^2}}{\theta} \right] \mp 1 \right\}^{-1} \frac{p^4 dp}{\sqrt{m^2+p^2}} \,.$$

# Global Thermodynamic Equilibrium (LTE)

ЛТР не обеспечивает отсутствие диссипативных процессов, поскольку локально-равновесные функции распределения (84), обращают в нуль интегралы столкновений, но не левые части кинетических уравнений (see 29).

**If put kinetic equations' left parts are equal to zero we obtain the conditions of global thermodynamic equilibrium (ГТР):**

$$\underset{\xi}{\mathbf{L}}\, g_{ik} = 0, \quad m_a \not\equiv 0; \tag{89}$$

$$\underset{\xi}{\mathbf{L}}\, g_{ik} = \sigma g_{ik}, \quad m_a \equiv 0, \tag{90}$$

где $\underset{\xi}{\mathbf{L}}\, \Psi$ – производная Ли от $\Psi$ по направлению $\xi$.

С учетом времениподобности вектора $\xi$ (см. (85)) это означает, что *в случае наличия в плазме массивных частиц ГТР возможно лишь в стационарных полях тяготения, а в случае, когда все частицы являются безмассовыми, ГТР возможно лишь в конформно - стационарных полях тяготения.*

**Strict cosmological consequence of Kinetic Theory:**

Применительно к космологии это, в свою очередь, означает, что строгое термодинамическое равновесие достижимо лишь в случае ультрарелятивистской плазмы.