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Multivalent Guiding Function in a Problem on Existence of Periodic Solutions of Some Classes of Differential Inclusions

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Abstract—We propose to use the multivalent guiding function for the study of periodic solutions of some classes of differential inclusions. More precisely, we consider the periodic problem for nonlinear systems described by differential inclusions with both convex and nonconvex right-hand side. The latter include differential inclusions with a regular right-hand side. Note that the class of regular multimaps is wide enough. It includes, for example, bounded almost lower semicontinuous multimaps with compact values.

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Introduction. Fundamentals of the method of guiding functions were founded in papers by M. A. Krasnosel'skii and A. I. Perov (for example, [1–3]). One of the most essential directions of its development on the case of differential equations is presented by the method of multivalent vector guiding functions by D. I. Rachinskii (for example, [4, 5]).

It is well-known that the use of methods of theory of topological degree to the solution of different problems of nonlinear analysis and theory of differential equations is very efficient [1, 2, 6–10].

A series of papers (for example, [6, 8, 11, 12]) is devoted to the expansion of the classical method of guiding potentials, including its nonsmooth analog, on the case of differential inclusions and its using for investigation of their periodic solutions.

Different modifications of the method of multivalent vector guiding functions were used in the problem about the existence of periodic solutions to differential inclusions in the case of convex right-hand side, only [13–15].

In the present paper, developing the approach proposed in [11], we propose to use a multivalent vector guiding function for the investigation of the problem about the existence of periodic solutions of some classes of differential inclusions. More precisely, we consider a periodic problem for nonlinear systems described by differential inclusions with both convex and nonconvex right-hand sides. The latter includes differential inclusions with a regular right-hand side. We note that the class of regular multimaps is rather wide. It contains, for example, bounded almost lower semicontinuous multimaps with compact values.

1. Preliminaries. In what follows we use the known notions and the terminology from the analysis and theory of multivalued mappings (multimaps) [6, 16–18]. Let us remind some of them.

Let E be a separable Banach space, $L^1([a,b];E)$ denote a Banach space of Bochner summable functions $f:[a,b]\to E$.

Definition 1. A nonempty set $M\subset L^1([a,b];E)$ is said to be decomposable, if for any $f,g\in M$ and any Lebesgue measurable set $m\subset[a,b]$ it holds $f\varkappa_m+g\varkappa_{([a,b]\setminus m)}\in M$, where $\varkappa_m$ is the characteristic function of the set m.

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