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GENERALIZED CONVOLUTIONS OF H-TRANSFORMS

V.A. Kakichev and Nguen Suan Thao

Generalized integral convolutions generated by three operators were first studied in [1] for transforms of the Mellin type (see [2]), then in [3] for G-transforms (see [4]), and in [5] for the transforms of Kontorovich–Lebedev type (see [6]). In [7] a constructive method was suggested to define the most general integral convolutions with weights which allow to construct new types of convolutions, among these the generalized integral convolutions generated by three Fourier transforms, cosine and sine transforms (see [8]). In the same paper an application of the convolutions to solving the systems of integral equations was given.

In this article we construct convolutions by three H-transforms and study their properties. It is well to observe that the convolution kernel uniquely determines all three H-transforms generating the convolution (Theorem 2). The convolutions are applied to solving the general systems of the convolution equations (Theorem 4) and examples of the systems are presented.

1. Generalized convolutions

Definition 1 (see [9]). We define H_k-transform as

$$F_k(x) = (H_k f_k)(x) = \frac{1}{2\pi i}\int_\sigma X^{p^k}_{\overline{m}^k,\overline{a}^k,\overline{\alpha}^k}(s) f_k^*(s) x^{-s} ds, \quad k = \overline{1,3}, \tag{1}$$

where

$$X^{p^k}_{\overline{m}^k,\overline{a}^k,\overline{\alpha}^k}(s) = \prod_{j=1}^{p^k} \Gamma^{m_j^k}(b_j^k + \alpha_j^k s), \quad b_j^k = \tfrac{1}{2} - (a_j^k - \tfrac{1}{2})\operatorname{sign}(\alpha_j^k),$$

$$m_j^k \in Z, \ \ a_j^k \in \mathbb{C}, \ \ \alpha_j^k \in \mathbb{R}, \ \ \overline{m}^k = (m_1^k, \dots, m_{p_k}^k), \ \ \overline{a}^k = (a_1^k, \dots, a_{p_k}^k), \ \ \overline{\alpha}^k = (\alpha_1^k, \dots, \alpha_{p_k}^k).$$

Here $\alpha^k + 1 > (2\operatorname{Re} a_j^k - 1)\operatorname{sign}(\alpha_j^k)$, $j = \overline{1, p^k}$. $f^*(s)$ is the Mellin transform (see [8]) of $f(x)$, $\sigma = \{s, \ \operatorname{Re} s = \frac{1}{2}\}$.

Definition 2. Generalized convolutions of H_k-transform (1) are defined as follows:

$$(f_i \overset{k}{*} f_j)(x_k) = \frac{1}{x_k}\int_0^{+\infty}\int_0^{+\infty} \Theta_k(x_k, s, t) f_i(s) f_j(t) ds\, dt, \tag{2}$$

where (see [10], [11])

$$\Theta_k(x_k, s, t) = \frac{1}{st} H\left(\begin{matrix} x_k/s \\ \\ x_k/t \end{matrix} \;\middle|\; \begin{matrix} p^i & \overline{m}^i & \overline{a}^i & \overline{\alpha}^i \\ p^j & \overline{m}^j & \overline{a}^j & \overline{\alpha}^j \\ p^k & -\overline{m}^k & \overline{a}^k & \overline{\alpha}^k \end{matrix}\right),$$
$$i, j, k = 1, 2, 3, \quad i \neq j, \quad j \neq k, \quad k \neq i.$$