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DUALITY IN SPACES OF POLYHARMONIC FUNCTIONS

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In this article we consider the questions of description of conjugate spaces for spaces of polyharmonic functions.

More exactly, let Δ be the Laplace operator in $\mathbb{R}^n$, $n \geq 2$, and D a bounded domain (i. e., an open connected set) in $\mathbb{R}^n$ with smooth boundary. A (complex-valued) function $u \in C^{2m}(D)$ will be called polyharmonic (more exactly, m-harmonic, $m \in \mathbb{N}$) in the domain D if $\Delta^m u = 0$ in D.

Let us denote by $\mathrm{sol}(\Delta^m, D)$ the space of functions m-harmonic in the domain D, which is equipped with the Frechet–Schwartz topology, i. e., the topology of the convergence uniform along with all derivatives on compact subsets of D.

We denote by $\mathrm{sol}(\Delta^m, D)'$ the space conjugate to $\mathrm{sol}(\Delta^m, D)$. Let us equip this space with the strong topology. i. e., the topology of the uniform convergence on bounded subsets of $\mathrm{sol}(\Delta^m, D)$.

As is known, any successful characterization of the dual space $\mathrm{sol}(\Delta^m, D)'$ gives us an additional information about solutions of the equation $\Delta^m u = 0$ (see for Golubev' series [1], for theorems on erasing of singularities [2] and so on).

Several classical representations of the conjugate space for $\mathrm{sol}(\Delta^m, D)$ exist, as, for example, the Grothendieck duality (see [3]), where an appropriate Green's formula is used. In this article the duality is obtained by means of the scalar product in the Lebesgue space $L^2(D)$.

First we will formulate the basic result.

Let $\mathrm{sol}(\Delta^m, \overline{D})$ stand for the space of functions m-harmonic in a neighborhood of the closure of the domain D, equipped with the standard topology of the inductive limit with respect to a certain decreasing sequence of neighborhoods of $\overline{D}$.

Theorem 1. *If the boundary of the domain D is connected and real-analytic, then* $\mathrm{sol}(\Delta^m, D)'$ *is topologically isomorphic to the space* $\mathrm{sol}(\Delta^m, \overline{D})$.

Stout (see [2]) obtained similar results for spaces of harmonic functions, by using the scalar product in the Hardy spaces for the construction of the duality.

For the spaces of holomorphic functions in simply-connected domains in $\mathbb{C}$ and (p, q)-circular domains in $\mathbb{C}^2$, a similar theorem was proved in [4]. Zorn (see [5]) obtained similar results for spaces of functions holomorphic in pseudoconvex domains in $\mathbb{C}^n$, using the scalar product in the space $L^2(D)$.

In [6], a similar theorem was proved for an arbitrary elliptic system A with coefficients real-analytic in domains which have real analytic boundaries and possess some properties of convexity with respect to the system A. However, in doing so, they used another duality.

Let us pass to the construction of the duality. To this end we suppose that the boundary of the domain D is connected, real-analytic (and smooth). Let $\rho(x)$ be the defining function of the domain D, i. e., a real-valued function defined in a certain neighborhood of $\overline{D}$ such that $|\nabla \rho| \neq 0$

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