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Two-Party Graphs and Monotonicity Properties of the Poincaré Mapping

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Abstract—The local differential of a system of nonlinear differential equations with a T-periodic right-hand side is representable as a directed sign interaction graph. Within the class of balanced graphs, where all paths between two fixed vertices have the same signs, it is possible to estimate the sign structure of the differential of the global Poincaré mapping (a shift in time T). In this case all vertices of a strongly connected graph naturally break into two sets (two parties). As appeared, the influence of variables within one party is positive, while that of variables from different parties is negative. Even having simplified the structure of a local two-party graph (by eliminating its edges), one can still exactly describe the sign structure of the differential of the Poincaré mapping. The obtained results are applicable in the mathematical competition theory.

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INTRODUCTION

Solving the problems of mathematical competition in variable medium is implicitly connected with the investigation of multi-dimensional, nonlinear and nonautonomic models. One of the effective solving approaches, i.e., the inheritance method, was introduced in [1] and is based on exact transfer of some local properties into global ones.

In the case of two rivals this allows us to establish global Poincaré map monotonicity properties. For three or more competitors, interaction monotonicity of the mapping appears only after time inversion trick application ([2], P. 91). Monotonicity property (from some semi-oder viewpoint) makes it possible to apply nonlinear analysis geometric methods ([3], P. 255).

Note that this inheritance principle is oriented onto pretty rough cases in which the mutual impact of all the variables does not vanish. In the non-rough case local interaction for some variables may vanish. This instance seriously hampers inheritance principle substantiation. For example, in [4] the author establishes inheritance of certain properties by some rather complicated limit transfer from the rough cases sequence to the non-rough one.

Nevertheless one can act in simplified manner by applying purely graphic geometric considerations from the graph theory and appropriate choice of the key solutions for the small initial time interval. This approach seems to be effective both in the rough and non-rough situations. This approach exposition is exactly the gist of the article.

So assume that we have a smooth (of the class C^∞) system of ordinary differential equations

$$\dot{x}_1 = f_1(X,t), \ldots, \dot{x}_n = f_n(X,t), \tag{1}$$

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