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ON LATTICE-UNIVERSAL VARIETIES OF ALGEBRAS

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1. Introduction and formulation of main results

If a lattice L is embeddable into the lattice of subalgebras of a certain algebra of a given class K, then we say that L *is representable by a lattice of subalgebras of the algebra from* K. The class K is said to be *lattice-universal* if any lattice is representable by the lattice of subalgebras of a certain algebra from K (the term "lattice-universal class" was suggested to the author by L.N. Shevrin). The classical Whitman theorem (see [1]) on representation of lattices by the lattices of subgroups ascertains in our terminology that the class of all groups is lattice-universal. Hence and from the fact that the lattice of subgroups of a group is a sublattice of the lattice of semisubgroups it follows automatically that the class of all semigroups is lattice-universal, too. Some proper lattice-universal subclasses in the class of semigroups, which stand far from the class of groups, were found by the author (see in [2]). Among those, we cite the class of all semilattices and the class of all commutative nilsemigroups of the index two.

The above-mentioned classes of algebras form varieties. This circumstance naturally leads to formulation of the following problem: *For the class of algebras of either this or that concrete signature, characterize all lattice-universal varieties of these algebras.* In the present article we consider this problem in the case of semigroups, associative rings, and lattices. The following three theorems represent the main results.

Theorem 1.1. *A variety of semigroups V is lattice-universal if and only if V satisfies one of the conditions*:

(1) *V contains a semigroup which is not nilpotent extension of the rectangular band of groups*;
(2) *V is a periodic variety and any lattice is representable by the lattice of semigroups of a certain group from V.*

Theorem 1.2. *Any non-nilpotent variety of associative rings is lattice-universal.*

Theorem 1.3. *The variety of lattices is lattice-universal if and only if it is nontrivial.*

Let us note that Theorem 1.3 is a strengthening of the corresponding result by Hong, who considered in [3] embeddings of finite lattices into lattices of sublattices.

As we can see, the question on a complete description of lattice-universal varieties of semigroups is reduced by Theorem 1.1 to a description of corresponding group periodic varieties. The latter problem seems to be very difficult. Until most recent time the author did not know any example of a proper non-Abelian variety of groups V, for which the answer to the question: Whether V is lattice-universal? were known. Certain movement along this direction was done by the author recently. So, for example, it was established that for arbitrary integer odd number $k \geq 665$ the variety of all groups of the exponent of k is lattice-universal. Moreover, it was shown that nilpotency of a group (ring) implies the fulfillment of a nontrivial identity in the corresponding lattice of subgroups