ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 2, pp. 75–89. © Allerton Press, Inc., 2011.
Original Russian Text © A.G. Chentsov, 2011, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2011, No. 2, pp. 86–102.

On Approximate Constraint Satisfaction

A. G. Chentsov[1*]

[1] *Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, ul. S. Kovalevskoi 16, Ekaterinburg, 620219 Russia*

Received June 8, 2009

Abstract—We consider an abstract problem on fulfilling asymptotic constraints. We propose a very general approach to constructing "nonsequential" attraction sets in the space of generalized elements formalizable as finitely additive measures. We study the existence and the structure of the asymptote universal in the range of "asymptotic constraints" not requiring the compactifiability of the space of ordinary solutions.

DOI: 10.3103/S1066369X11020095

Keywords and phrases: *bitopological space, finitely additive measure, extension.*

1. INTRODUCTION

We use the following abbreviations: BTS (bitopological space), f.-a. (finitely additive), AS (attraction set), AD (accessibility domain), GC (generalized control), and TS (topological space).

This paper is dedicated to the construction of correct extensions of abstract control problems which lose the stability under relaxed constraints. In [1] (Chap. 3) one has introduced the notions of exact, (sequential) approximate, and generalized controls. In this paper we consider the latter two types of controls, identifying approximate controls with directionalities, and doing generalized ones with f.-a. measures. In connection with the use of directionalities, note remarks in papers [2] and [3]. Control-measures were widely used for realizing sliding regimes [1, 4–7]; f.-a. measures were used as GC in [8–11] and some other papers.

The considered problem can be connected with some control problems for linear systems under integral constraints which include the moment component (the "occurrence " of impulse constraints is also possible) ([8], § 6.5; [9], Chaps. 1, 2). A relaxation of these constraints can cause an abrupt improvement of process characteristics. This is connected with the appearance of new controls feasible under relaxed constraints. The latter fact has a special interest, but it also plays the key role in studying the potentially attainable quality in the case most interesting from the practical point of view, when constraints are fulfilled with a great but finite degree of exactness (monographs [8–11]). Now we restrict ourselves to the analysis of the simplest scalar system

$$\dot{x}(t) = b(t)f(t), \;\; 0 \le t \le 2, \;\; x(0) = 0. \tag{1.1}$$

Here $b = b(\cdot)$ is a real-valued function on the segment $[0, 2[$, for which $b(t) \triangleq t$ with $t \in [0, 1[$ and $b(t) \triangleq t - 1$ with $t \in [1, 2[$ (hereinafter the symbol $\triangleq$ means the equality by definition), and $f \in F$, where F is the set of all real-valued nonnegative piecewise constant and right-continuous functions on $[0, 2[$ such that for each one of them the integral on $[0, 1[$ does not exceed one. Assuming that $\varphi_f = (\varphi_f(t), \, 0 \le t \le 2)$ is the trajectory of system (1.1) generated by the control $f \in F$, we impose the intermediate condition $1 \le \varphi_f(1)$. We are interested in the AD at the final time moment, namely, $G_\partial \triangleq \{\varphi_f(2) : f \in F, 1 \le \varphi_f(1)\}$. Evidently, $G_\partial = \emptyset$, because

$$\varphi_f(1) = \int_0^1 b(t)f(t)dt = \int_0^1 tf(t)dt < 1 \;\; \forall f \in F. \tag{1.2}$$

*E-mail: chentsov@imm.uran.ru.