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SKEW-SYMMETRIC ITERATION METHODS FOR SOLVING STATIONARY PROBLEM OF CONVECTION-DIFFUSION WITH A SMALL PARAMETER AT THE HIGHEST DERIVATIVE

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1. Introduction

The presence of a small parameter at the highest derivative in the equation of convection-diffusion is equivalent to the prevailing of convective terms of the equation over diffusion ones and this complicates the numerical solving. Under certain conditions such a situation corresponds to arising of a boundary layer, i.e., to the strong growth at a narrow part of the domain of calculation. Thus phenomenon is characteristic for a wide range of problems (see [1]). Approximation of similar problems by finite differences leads to necessity to solve strongly asymmetric systems of linear algebraic equations.

By *strongly asymmetric systems of linear algebraic equations*

$$Au = f, \quad A = A_0 + A_1, \quad A_0 = A_0^*, \quad A_1 = -A_1^*, \tag{1}$$

we shall understand the systems, whose matrices' symmetric part is enough lesser (in the sense of a certain norm) than the skew-symmetric part.

However, the presence of a small parameter at the highest derivative is a necessary but not sufficient condition for appearing of a boundary layer in a problem to solve. Appearance of the boundary layer essentially depends on both the boundary value conditions and the right side of the initial differential equations, i.e., on the properties of the function f in (1), which contains these data after approximation of the differential equation.

Consider in the domain Ω the following boundary value problem:

$$-\frac{1}{Pe}\Delta U + v_1\frac{\partial U}{\partial x} + v_2\frac{\partial U}{\partial y} = f(x,y), \qquad U|_{\partial\Omega} = U_{\mathrm{bndr}}. \tag{2}$$

The most complicated stage of the approximating problem (2) is related to the choice of an approximation for the convective terms of the equation, though the number of variants is rather bounded, because there can be used either the first order approximation (of difference "forward" or "back"), or the central-difference approximation of the second order of exactness.

It is known (see [2]) that in using differences "forward" or "back" one must keep in difference form the properties of the monotonicity and maximum principle, which, in transforming the difference scheme into a system of linear algebraic equations, give in (1) an M-matrix (see [3]). (If signs of the functions $v_1(x,y)$ and $v_2(x,y)$ are not constant, then instead of the differences "forward" or "back" there are used the differences "counter-flow".) For this case, there can be suggested a wide collection of iteration methods which solve effectively systems with the M-matrices (see [4]) and, first of all, the methods based on incomplete decomposition by Kholetskiĭ in combination with gradient methods (see [5]). But the low exactness of the first order schemes (especially in the case