

PROBLEM OF CONJUGATING SOLUTIONS OF THE LAME EQUATION IN THE AREAS WITH PIECEWISE SMOOTH BOUNDARIES

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The solution of two-dimensional problems of conjugating the analytical functions in the given and affine transformed areas with piecewise smooth boundaries [1], [2] enable one to solve a number of applied problems of continuum mechanics [3]–[9]. In this paper, we investigate the problem of conjugating solutions of the Lame equations [10] in three-dimensional areas with piecewise smooth boundaries.

Problem definition. Let the three-dimensional space R_3 contain a simply connected finite domain V_1 which is bounded by a certain piecewise smooth surface S . The latter is supposed to contain disjoint smooth closed singular lines (sets of extreme points) and the conic points which do not belong to them.

We construct the vector functions $\bar{u}_i(M) = \{u_{1i}(M), u_{2i}(M), u_{3i}(M)\}$ ($i = \overline{0,1}$, $M = M(x_1, x_2, x_3)$) defined in the corresponding domains V_i , where $V_0 = R_3 \setminus V_1$, and satisfying the Lame equation [10]

$$(\lambda_i + 2\mu_i) \operatorname{grad} \operatorname{div} \bar{u}_i(M) - \mu_i \operatorname{rot} \operatorname{rot} \bar{u}_i(M) = 0, \quad (1)$$

where $\lambda_i = \frac{2\nu_i\mu_i}{1-2\nu_i}$, $0 < \nu_i < 0.5$; $\mu_i \in (0, \infty)$, $\mu_i \neq \infty$,

and the condition of conjugation on the surface S of the boundary of the areas V_0 and V_1 : at the points of smoothness

$$\bar{v}_0^-(M) - \bar{u}_1^+(M) = 0, \quad N_{n0}^-[\bar{u}_0(M)] - N_{n1}^+[\bar{u}_1(M)] = 0, \quad (2)$$

at the singular points M^0

$$\lim_{M \rightarrow M^0} (\bar{u}_0^-(M) - \bar{u}_1^+(M)) = 0, \quad \lim_{M \rightarrow M^0} (N_{n0}^-[\bar{u}_0(M)] - N_{n1}^+[\bar{u}_1(M)]) = 0, \quad (3)$$

where the operator $N_{ni}[\cdot]$ acts due to the rule

$$N_{ni}[\bar{u}_i(M)] = 2\mu_i \frac{\partial \bar{u}_i(M)}{\partial n} + \lambda_i \bar{n} \operatorname{div} \bar{u}_i(M) + \mu_i [\bar{n}, \operatorname{rot} \bar{u}_i(M)],$$

\bar{n} is a normal to the surface S , exterior with respect to the area V_1 ; $\bar{u}_i^\pm(M)$, $N_{ni}^\pm[\bar{u}_i(M)]$, $i = \overline{0,1}$, are the boundary values of the vector functions $\bar{u}_i(M)$, $N_{ni}[\bar{u}_i(M)]$, when approaching the surface S from the side of the area V_1 (“+” sign) or V_0 (“−” sign), $[\dots, \dots]$ is the vector product notation.

At the points of the singular lines or at the conic points, we understand the implementation of the Lame equations in the sense of the equality

$$\lim_{\Delta V \rightarrow M^0} \iint_{\Delta V} ((\lambda_i + 2\mu_i) \operatorname{grad} \operatorname{div} \bar{u}_i(x_1, x_2, x_3) - \mu_i \operatorname{rot} \operatorname{rot} \bar{u}_i(x_1, x_2, x_3)) dv = 0. \quad (4)$$