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ON THE INTRINSIC GEOMETRIES OF THE MANIFOLD OF PLANAR GENERATORS OF SIX-DIMENSIONAL QUADRIC

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We shall study a Riemannian connection in the tangent bundle $\tau(CV_6)$ of a 6-dimensional manifold CV_6, where CV_6 is a complex analytic Riemannian space. With the use of technique developed in [1] we construct a prolongation of this connection into the spinor bundle $A^C(CV_6)$, i.e., into a vector bundle whose fiber is isomorphic to C^4. This special case is of interest because of its applications to the theory of relativity (see [2], Vol. 2, p. 355).

Let us consider a nondegenerate quadric CQ_6 embedded into the projective space CP_7. This quadric can be defined via the equation

$$G_{AB}X^AX^B = 0 \;\Leftrightarrow\; (X,X) = 0 \quad (A,B,\ldots = \overline{1,8}). \tag{1}$$

By the Cartan triality principle (see [3], p. 159), the manifold of points of the quadric is diffeomorphic to the manifold of three-dimensional planar generators, which constitute two families. The base points of the planar generators

$$X_a = ({X_a}^A) \quad (a,b,\ldots,i,j,\ldots,p,q,\ldots = \overline{1,4}) \tag{2}$$

satisfy equation (1)

$$(X_a, X_b) = 0. \tag{3}$$

Let us define the planar generator by its matrix coordinate $Z = (Z^p_a)$ (see [4]):

$$X_a := A_a + B_pZ^p_a, \quad (A_a, B_p) := d_{ap}, \quad B^a := d^{ap}B_p, \tag{4}$$

then from (3) it follows

$$Z_{ab} = -Z_{ba}, \quad Z_{ab} := d_{ap}Z^p_a. \tag{5}$$

This means that X_a depends on the six complex parameters. Consider the manifold M of planar generators of maximal dimension of CQ_6. We denote by RM the real representation of M. As it is known (see [1]), the normalization of RM is determined by a real differentiable correspondence between the planar generators of maximal dimension of CQ_6:

$$f : CP_3(X_a) \to CP_3(Y_p), \tag{6}$$

which is defined as follows: To a planar generator $CP_3(X_a)$ one must put in correspondence the plane $CP_3(Y_p)$ which does not meet $CP_3(X_a)$. For the six-dimensional quadric these planar generators need to lie in the same family. Then the bundle $A^C(RM)$ can be constructed in the following way. For the base we take RM, and for the fiber over a point in RM we take, by the triality principle, the vector space C^4 which represents the planar generator of maximal dimension of cone