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ESTIMATES OF CONDITIONAL STABILITY OF A TWO-CONTOUR BOUNDARY VALUE PROBLEM IN A CIRCLE

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1. In [1]–[3] a different-contour boundary value problem for the polyharmonic equation

$$\Delta^n u + c_1\Delta^{n-1}u + \cdots + c_n u = 0 \qquad (1_{2n})$$

was considered with the Laplace operator Δ and constant coefficients $c_1, \dots, c_n$, when the boundary value conditions were given on the different contours

$$u(x)\big|_{\Gamma_1} = \varphi_1(x), \dots, \quad u(x)\big|_{\Gamma_n} = \varphi_n(x), \qquad (2_{2n})$$

where the closed contours Γ_k bound the domains D_k; moreover, $D_1 \subset D_2 \subset \cdots \subset D_n \subseteq D$. In addition, as it was shown in [2], [3], the results will not change practically if one takes instead of the Laplace operator Δ the Laplace–Beltrami operator L and consider the boundary value problem (2) for the corresponding equations in the spaces of constant curvature. Moreover, let us note that problems of that kind are a possible generalization of multi-point problems for ordinary differential equations and arise in various applied problems, related, e. g., to geological, geophysical and other observations (see [4]), where the measurement of experimental data is carried out only on the boundary of a domain, but not inside it.

Below we shall show that problem (1_{2n}), (2_{2n}) is ill-posed, therefore an important role is played by estimation of the conditional stability of the problem. Here the estimates for the problem of analytic Hadamard continuation (see [4], pp. 26–27) can be referred as classic. In the present article we also dwell into estimation of the conditional stability for problem (1_{2n}), (2_{2n}) for $n = 2$, i. e., we consider the fourth order equation

$$K_4 u = \Delta^2 u + c_1\Delta u + c_2 u = 0, \qquad (1_4)$$

when the contours Γ_1 and Γ_2 have the form of concentric circumferences with the radii R_1 and R_2, respectively, $R_2 > R_1$. (In further notation in (1), (2) the subscript 4 will be omitted when it makes no confusion.)

Equation (1) can be rewritten via the roots of the square equation

$$\lambda^2 + c_1\lambda + c_2 = 0 \qquad (3)$$

in the form of a superposition of two operators of second order $K_4 u = (\Delta - \lambda_2)(\Delta - \lambda_1)u = 0$ or an equivalent system of second order equations

$$\begin{cases} (\Delta - \lambda_1)u = v, \\ (\Delta - \lambda_2)v = 0. \end{cases} \qquad (4)$$

By virtue of (4), an arbitrary solution of (1) can be represented in the form

$$u = u_1 + \widetilde{v}, \qquad (5)$$