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On Definability of Completely Decomposable Torsion-Free Abelian Groups by Certain Groups of Homomorphisms

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Abstract—Let C be an Abelian group. An Abelian group A from a class X of Abelian groups is said to be ${}_CH$-definable in X if, for any group $B \in X$, the isomorphism $\mathrm{Hom}(C, A) \cong \mathrm{Hom}(C, B)$ implies that $A \cong B$. If every group from X is ${}_CH$-definable in X, then X is called an ${}_CH$-class. In this paper, we study conditions under which a class of completely decomposable torsion-free Abelian groups is an ${}_CH$-class, where C is a vector group.

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The well-known Problem 34 of L. Fuchs [1] has been solved by P. Hill [2] and A. M. Sebel'din [3], namely, it has been proved that there exists no Abelian group C such that, for any groups A and B, $\mathrm{Hom}\,(A, C) \cong \mathrm{Hom}\,(B, C)$ implies that $A \cong B$. What is more, in [3], completely decomposable nonisomorphic Abelian groups A and B were found such that $\mathrm{Hom}\,(A, C) \cong \mathrm{Hom}\,(B, C)$ for any Abelian group C. In this paper, we consider the inverse problem.

Let X and Y be some classes of Abelian groups. We will say that Y is an ${}_CH$-class for a group $C \in X$ if from $\mathrm{Hom}\,(C, A) \cong \mathrm{Hom}\,(C, B)$ it follows that $A \cong B$ for any $(A, B) \in Y \times Y$.

In [4], necessary and sufficient conditions have been found under which a completely decomposable torsion-free Abelian group C is such that the class of completely decomposable torsion-free Abelian groups is an ${}_CH$-class. In this paper, we solve a similar problem in the case when C is a vector group.

By a vector group we mean a torsion-free Abelian group decomposable into a direct product of rank one groups: $V = \prod\limits_{i \in I} R_i$.

All groups under consideration in the paper are torsion-free Abelian groups. We use the following notation: $r\,(A)$ is the rank of a group A, $\tau\,(A)$ denotes the of a rank one torsion-free Abelian group, Ω is the set of various types of rank one torsion-free Abelian groups, $\Omega\,(A)$ is the set of various types of rank one direct summands in a torsion-free Abelian group A, $\Omega_0\,(A)$ is the set of all types from $\Omega\,(A)$ whose characteristics do not contain the symbol ∞.

Theorem. *Let C belong to the class F_v of vector groups. The class F_{cd} of completely decomposable torsion-free Abelian groups is an ${}_CH$-class if and only if C satisfies the following conditions*:

1. *C contains a direct summand isomorphic to Z*;
2. *$\Omega\,(C)$ contains only idempotent types*;
3. *C is a group of finite rank.*

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