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ON A METHOD OF OBTAINING AN EXPONENTIAL ESTIMATE FOR THE SOLUTION OF VOLTERRA EQUATION

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The proposed method is a successive performance of the following steps.

1. The matrix kernel of the equation

$$x(t) = \int_0^t K(t,s)x(s)ds + f(t), \quad t \in [0,\infty[, \tag{1}$$

having an exponential order of growth, is replaced by a matrix

$$\overline{K}(t,s) \stackrel{\text{def}}{=} K(t,s)\exp(\beta(t-s)), \tag{2}$$

whose order of growth is more than a positive number to be given below.

2. Construct a matrix $Q(\cdot,\cdot)$ which is the principal part of the kernel $\overline{K}(\cdot,\cdot)$, so that an exponential estimate of its resolvent can be found with the required accuracy, and the resulting additional term

$$H(t,s) \stackrel{\text{def}}{=} \overline{K}(t,s) - Q(t,s)$$

does not essentially influence on it. Thus, the estimate for the resolvent of the kernel $\overline{K}(\cdot,\cdot)$ appears to be known.

3. We use (2) to determine an exponential estimate for the resolvent of $K(\cdot,\cdot)$.

4. The required estimate is found by means of the formula for solving equation (1)

$$x(t) = \int_0^t R^K(t,s)f(s)ds + f(t), \quad t \in [0,\infty[. \tag{3}$$

The method is essentially based on the associated matrix product introduced by V.R. Vinokurov in [1]. In particular, it can be applied to equation (1), whose kernel is almost periodic with respect to both variables; in this case we construct a "close" to $\overline{K}(\cdot,\cdot)$ periodic matrix $Q(\cdot,\cdot)$, whose resolvent can be estimated with any required accuracy (see [2]).

In this brief communication we present Theorem 2 which enables us to construct the required $Q(\cdot,\cdot)$ on the first stage, thus making it unnecessary to construct repeatedly the matrix, then to find an exponential estimate for its resolvent and afterwards to apply it to the estimate of the additional term. This is especially burdening when the exponential estimate for the resolvent of the kernel $Q(\cdot,\cdot)$ is associated with significant, although technical, difficulties. However, if the resolvent of $Q(\cdot,\cdot)$ can be easily estimated, then Theorem 1 can be applied.

In what follows $\|\cdot\|$ is a norm in an n-dimensional vector space, $\|A\|$ is the consistent norm of $n\times n$-matrix A; $R^K(\cdot,\cdot)$ is the resolvent of the kernel $K(\cdot,\cdot)$; $\Delta \stackrel{\text{def}}{=} \{(t,s) : 0 \le s \le t < \infty\}$. When writing the relations between summable functions, we shall omit the words "almost everywhere".

Recall (see [3]) that an $n\times n$-matrix $K(\cdot,\cdot) = (k_{ij})$, defined in Δ, satisfies the local condition $\mathcal{U}$, if for any $b > 0$ there holds at least one of the inequalities: