Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 47, No. 9, pp.50–54, 2003 *UDC 517.958:532.5*

THE ACCURACY OF APPROXIMATE SYMMETRIES OF EQUATIONS DESCRIBING THE DYNAMICS OF THE NON-NEWTONIAN FLUID AND INVARIANT SOLUTIONS OF THESE EQUATIONS. II

L.D. Eskin and S.L. Tonkonog

The present paper is a continuation of [1], where we studied conditions under which the equations describing dynamics of surface of non-Newtonian fluid with the Ostwald – de Waele power rheological law are exactly invariant. Here, using the methods of qualitative theory of dynamic systems (see [2]), we study invariant solutions of these equations. Also, we demonstrate how to apply our results of qualitative analysis to solving boundary value problems with invariant solutions.

We use the notation of [1], and keep enumeration of Items and formulas.

4. We will apply the qualitative analysis of planar dynamic systems (see [2], where many examples are given; for other examples we refer the reader to [3], [4]) to equation (2.17). To this end, we investigate singular points of this equation, its isocline for zero and for infinity, asymptotic of integral curves in a neighborhood of singular points and at infinity. Let $V = d_2 - pd_1$. If $(d_1, d_2) \notin V = 0$, then we have two singular points: $O(0,0)$ and $T(z_1, 0)$, $z_1^{2n+1} = -rp^{-n}d_2$. Using standard technique of studying singular points (see [2]), we find that $O(0,0)$ is a non-simple saddle-knot with unstable knot sector, the integral curves leave the point O in the direction with slope $k = -d_2/d_1$. If $(d_1, d_2) \in V = 0$, then the singular point disappears and (2.17) takes the form

$$\frac{dy}{dz} = \frac{P_1(z,y)}{Q_1(z,y)},$$
$$P_1 = y[(n+2)z^{n+1}w^{n-1} + npz^{n+2}w^{n-2}] + r^{-1}z^{n+2}w^{n-1} + d_1,$$
$$Q_1 = -nyz^{n+2}w^{n-2}, \quad w = y + pz.$$

We set

$$a_n = \frac{(7n^2+10n+4)r}{p(2n+1)^2}, \quad b_n = \frac{nrq}{p}, \quad \lambda_{1,2} = (a_n \pm 2\sqrt{b_n})q^{-1} > 0.$$

T is a simple knot if $\frac{d_1}{d_2} < \lambda_2$ or $\frac{d_1}{d_2} > \lambda_1$, T is a simple focus if $\lambda_2 < \frac{d_1}{d_2} < \lambda_1$. The trajectories enter the knot (and leave it) in directions with slopes

$$K_{1,2} = (2b_n)^{-1}\left(\frac{d_1}{d_2}q - a_n \pm \sqrt{(a_n - \tfrac{d_1}{d_2}q)^2 - 4b_n}\right), \quad 0 > K_1 > K_2.$$

At the infinity we have a simple saddle in the direction with slope $K_3 = -\frac{1}{2(n+1)r}$ and non-simple knots in the direction of the line $z = 0$, $w = 0$. When studying the isocline for zero, which is an algebraic curve $P(z,y) = 0$, one has to take into account that this curve meets the coordinate axes (in case $(d_1, d_2) \notin V = 0$) at the points O and T, and does not intersect the straight line $w = 0$ at

The work was supported by the Russian Foundation for Basic Research, grant no. 00-01-00128.