ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2008, Vol. 52, No. 4, pp. 38–52. © Allerton Press, Inc., 2008.
Original Russian Text © A.G. Kushner, 2008, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2008, No. 4, pp. 43–58.

Contact Linearization of Nondegenerate Monge–Ampère Equations

A. G. Kushner[1*]

[1] *Astrakhan State University, ul. Tatischeva 20a, Astrakhan, 414056 Russia;*
Institute of Control Sciences, Russian Academy of Sciences,
ul. Profsoyuznaya 65, Moscow, 117997 Russia

Received May 31, 2007

Abstract—The present paper is devoted to the problem of transforming the classical Monge–Ampère equations to the linear equations by change of variables.
The class of Monge–Ampère equations is distinguished from the variety of second-order partial differential equations by the property that this class is closed under contact transformations. This fact was known already to Sophus Lie who studied the Monge–Ampère equations using methods of contact geometry. Therefore it is natural to consider the classification problems for the Monge–Ampère equations with respect to the pseudogroup of contact transformations.
In the present paper we give the complete solution to the problem of linearization of regular elliptic and hyperbolic Monge–Ampère equations with respect to contact transformations.
In order to solve this problem, we construct invariants of the Monge–Ampère equations and the Laplace differential forms, which involve the classical Laplace invariants as coefficients.

DOI: 10.3103/S1066369X08040051

Key words and phrases: *contact transformations, tensor invariants, Laplace invariants.*

1. INTRODUCTION

During the last one and a half century, the Monge–Ampère equations attracted attention of geometers. This can be explained by the fact that the Monge–Ampère equations which arise in solving geometrical problems are geometrical objects themselves.

The classical Monge–Ampère equation is

$$Av_{xx} + 2Bv_{xy} + Cv_{yy} + D(v_{xx}v_{yy} - v_{xy}^2) + E = 0, \tag{1}$$

where A, B, C, D, and E are functions depending on the variables x and y, the function $v = v(x, y)$, and its first derivatives v_x and v_y.

The class of Monge–Ampère equations is distinguished from the variety of second-order partial differential equations by the property that this class is closed under contact transformations. Note that the linear equations do not have this property. This fact was known already to Sophus Lie who studied the Monge–Ampère equations using methods of contact geometry [1–3]. In 1870–1880 he posed the problem to classify the Monge–Ampère equations with respect to the (pseudo)group of contact transformations, in part, to transform the Monge–Ampère equations to the quasilinear form (this means that $D = 0$ in (1)), and to find the simplest coordinate expression for these equations.

Sophus Lie considered also the classification problems for the Monge–Ampère equations which have intermediate integrals. G. Darboux and E. Goursat obtained important results in this direction for the equations of hyperbolic type. Sophus Lie himself found conditions for a Monge–Ampère equation to be reduced to the quasilinear form and to the linear form with constant coefficients. However he never published proofs of these theorems. Note that it is a rather complicated problem to verify whether a general Monge–Ampère equation has intermediate integrals.

The problem of reducibility of the nondegenerate Monge–Ampère equations to the equations with constant coefficients by the symplectic transformations was solved by V. V. Lychagin and V. N. Rubtsov

*E-mail: kushnera@mail.ru.