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Distributive and Neutral Elements of the Lattice of Commutative Semigroup Varieties

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Abstract—We completely describe commutative semigroup varieties that are distributive, standard, or neutral elements of the lattice of all commutative semigroup varieties. In particular, we prove that the properties of being a distributive element and of being a standard element in this lattice are equivalent.

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1. INTRODUCTION AND STATEMENTS OF MAIN RESULTS

The lattice of all semigroup varieties (we denote it by **SEM**) has been intensively studied for the last four decades. See the recent paper [1] for the systematic description of relevant results.

In the theory of lattices the noticeable attention is being paid to studying special elements of various types. Let us recall definitions of some of them necessary in what follows. An element x of a lattice $\langle L; \vee, \wedge\rangle$ is called
distributive, if

$$\forall y, z \in L \;\; x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z);$$

standard, if

$$\forall y, z \in L \;\; (x \vee y) \wedge z = (x \wedge z) \vee (y \wedge z);$$

modular, if

$$\forall y, z \in L \;\; y \leq z \longrightarrow (x \vee y) \wedge z = (x \wedge z) \vee y;$$

lower-modular, if

$$\forall y, z \in L \;\; x \leq y \longrightarrow x \vee (y \wedge z) = y \wedge (x \vee z);$$

neutral, if for any elements $y, z \in L$ elements x, y, and z generate a distributive sublattice in L.

Codistributive, costandard, and *upper-modular* elements are defined as dual to distributive, standard, and lower-modular ones, respectively. See, for example, [2], § III.2, for the vast information on [co]distributive, [co]standard, and neutral elements, which proves the natural character and the importance of studying them.

In recent years special elements of the lattice **SEM** have been studied in many papers. See [1], § 14, for their review. The lattice **SEM** contains many ample and important sublattices ([1], § 1, and Chap. 2). It is natural to study special elements of not only the whole lattice **SEM**, but also of its sublattices. One of the main sublattices in **SEM** is the lattice of all commutative semigroup varieties; we denote it by **Com**. The structure of this lattice is rather intricate. It suffices to say that it contains an isomorphic copy of

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