Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 7, pp.36–48, 1997 *UDC 517.518*

ON ESTIMATION OF THE STRONG MAXIMAL FUNCTIONS

A.K. Lerner

1. Introduction

In the present article we study estimates of the strong maximal Hardy-Littlewood and Fefferman-Stein functions.

By an *interval* in $\mathbb{R}^N$ we shall call a set of form

$$I = \{x : a_i \le x_i \le a_i + h_i,\ i = 1, \dots, N\}\ (h_i > 0);$$

given $h_1 = \dots = h_N$, the interval I is called a *cube*.

Let $f \in L^1_{\mathrm{loc}}(\mathbb{R}^N)$. Write $f_I = \frac{1}{|I|}\int_I f(t)dt$. Via the equalities

$$M_{\mathrm{st}} f(x) = \sup_{I \ni x} \frac{1}{|I|} \int_I |f(y)|dy \tag{1.1}$$

and

$$f^{\#}_{\mathrm{st}} f(x) = \sup_{I \ni x} \frac{1}{|I|} \int_I |f(y) - f_I|dy \tag{1.2}$$

there are defined the strong maximal Hardy-Littlewood and Fefferman-Stein functions, respectively (the upper bound is taken over all intervals $i \subset \mathbb{R}^N$ which contain the point x). If in (1.1) and (1.2) one takes the upper bound only over the cubes containing the point x, then one obtains ordinary maximal functions $Mf(x)$ and $f^{\#}(x)$ (see [1], [2]).

In [3] an estimate for the function $f^{\#}_{\mathrm{st}}$ was obtained in the terms of partial modules of continuity of the function $f \in L^p(\mathbb{R}^N)$ for $p > 1$. We shall give some definitions before starting to formulate this result.

Let $f \in L^p(\mathbb{R}^N)$ $(1 \le p < \infty)$. By a *partial module of continuity* of a function f with respect to a variable x_j $(1 \le j \le N)$ we call the function

$$\omega^{(j)}_p(f;\delta) = \sup_{0 \le h \le \delta} \left(\int_{\mathbb{R}^N} |f(x) - f(x + he_j)|^p dx \right)^{1/p} \quad (0 \le \delta < \infty),$$

where e_j is a vector whose j-th coordinate equals 1 while other ones equal zero. By a *module of continuity* we shall call every non-decreasing, continuous, bounded, and semiadditive on $[0, \infty)$ function $\omega(\delta)$ with $\omega(0) = 0$.

If $\omega_1(\delta), \dots, \omega_N(\delta)$ are the modules of continuity and $1 \le p < \infty$, then we denote by $H^{\omega_1,\dots,\omega_N}_p$ the class of all functions $f \in L^p(\mathbb{R}^N)$ such that for every function there holds

$$\omega^{(j)}_p(f;\delta) \le c\,\omega_j(\delta) \quad (j = 1, \dots, N;\ 0 \le \delta < \infty). \tag{1.3}$$