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Spectral Properties of Difference and Differential Operators in Weighted Spaces

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Abstract—We describe the spectrum of a difference operator (a weighted shift operator). We obtain applications to finding spectra of differential operators in weighted function spaces.

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Let X stand for a finite-dimensional linear normed space and let $LB(X)$ do for the algebra of linear operators acting in X. Denote by the symbol $l_\alpha^p = l_\alpha^p(\mathbb{Z}, X)$, where $p \in [1, \infty]$, the Banach space of (two-sided) sequences $x : \mathbb{Z} \to X$ of vectors from X summable with a weight (a weight function) $\alpha : \mathbb{Z} \to (0, \infty)$, with the norm

$$\|x\| = \|x\|_{p,\alpha} = \left(\sum_{n\in\mathbb{Z}} \left(\frac{\|x(n)\|}{\alpha(n)} \right)^p \right)^{1/p}, \quad p \in [1, \infty),$$

and bounded with respect to α

$$\|x\| = \|x\|_{\infty,\alpha} = \sup_{n\in\mathbb{Z}} \frac{\|x(n)\|}{\alpha(n)}, \quad p = \infty.$$

If $\alpha = 1$, then the space $l_\alpha^p(\mathbb{Z}, X)$ is denoted by $l^p = l^p(\mathbb{Z}, X)$.

In what follows we assume that the weight function α satisfies the condition

$$\sup_{n\in\mathbb{Z}} \frac{\alpha(n-1)}{\alpha(n)} < \infty. \tag{1}$$

Condition (1) is equivalent to the boundedness of the following difference operator (a weighted shift operator):

$$\mathcal{K} : l_\alpha^p \to l_\alpha^p, \quad (\mathcal{K}x)(n) = Bx(n-1), \quad n \in \mathbb{Z},\ x \in l_\alpha^p,$$

where $B \in LB(X)$. In this paper we study this operator and describe its spectrum.

The main result of the paper (Theorem 1) is obtained with the use of the values

$$\varkappa_{\text{out}}(\alpha) = \lim_{n\to\infty} \left(\sup_{k\in\mathbb{Z}} \frac{\alpha(k)}{\alpha(k+n)} \right)^{1/n}, \tag{2}$$

$$\varkappa_{\text{int}}(\alpha) = \lim_{n\to\infty} \left(\inf_{k\in\mathbb{Z}} \frac{\alpha(k)}{\alpha(k+n)} \right)^{1/n}, \tag{3}$$

constructed for the weight $\alpha : \mathbb{Z} \to (0, \infty)$. Note that limits in (2), (3) exist and $\varkappa_{\text{int}}(\alpha) \leqslant \varkappa_{\text{out}}(\alpha)$. If $\varkappa_{\text{int}}(\alpha) = \varkappa_{\text{out}}(\alpha)$, then the weight α is called balanced, and if $\varkappa_{\text{int}}(\alpha) = \varkappa_{\text{out}}(\alpha) = 1$, then it is called

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