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Properties of Renormalization Group Flow in the Neighborhood of Gaussian Fixed Point

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Abstract—We investigate the dynamics of the renormalization group transformation in the fermionic hierarchical model, given by the Lagrangian. We construct the invariant neighborhood of the Gaussian fixed point in the upper half-plane $g > 0$. We describe the subsets of the points of this neighborhood tending to the Gaussian fixed point under the iterations of the renormalization group transformation from the left and from the right.

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A hierarchical lattice Λ is defined as the set of integers Z with the hierarchical distance $d(i,j)$, $i,j \in N$, where $d(i,j) = n^{s(i,j)}$, if $i \neq j$; $s(i,j) = \min\{s$: there is a k such that $i \in V_{k,s}$, $j \in V_{k,s}\}$; $V_{k,s} = \{j : j \in N, (k-1)n^s < j \leq kn^s\}$, and n is the size of a unit cell (a fixed positive integer). When $n = p^d$, where p is prime number, the lattice Λ can be interpreted as lattice a of purely fractional d-dimensional p-adic vectors (see [1, 2]) with the p-adic distance between them. To each vertex i of the lattice Λ a four-component $\psi^*(i) = (\overline{\psi}_1(i), \psi_1(i), \overline{\psi}_2(i), \psi_2(i))$ of elements of the Grassmann algebra is assigned. All of these elements are generators of Grassmann algebra. A fermionic model on a hierarchical lattice is defined by the Hamiltonian

$$H(\psi^*;\alpha) = \sum_{i,j\in\Lambda} d_n^{-\alpha}(i,j)[\overline{\psi}_1(i)\psi_1(j) + \overline{\psi}_2(i)\psi_2(j)] + \sum_{i\in\Lambda} L(\psi^*;r,g),$$

where

$$L(\psi^*(i);r,g) = r(\overline{\psi}_1(i)\psi_1(i) + \overline{\psi}_2(i)\psi_2(i)) + g\overline{\psi}_1(i)\psi_1(i)\overline{\psi}_2(i)\psi_2(i).$$

To pass to a representation of the model in a projective space, we will use the concept of Grassmann-valued "density" of the free measure $f(\psi^*) = \exp\{-L(\psi^*;r,g)\}$. In the general case, the "density" of the free measure is defined as

$$f(\psi^*;c) = c_0 + c_1(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2) + c_2\overline{\psi}_1\psi_1\overline{\psi}_2\psi_2$$

with $c = (c_0, c_1, c_2) \in R^3$. In the regular case, when $c_0 \neq 0$, the coordinates r, g and c are related by formulas

$$r(c) = -\frac{c_1}{c_0}, \quad g(c) = \frac{c_1^2 - c_0c_2}{c_0^2}.$$

If $c_0 = 0$, which is the case, for example, when the density is defined by the Grassmann δ-function $\delta(\psi^*) = \overline{\psi}_1\psi_1\overline{\psi}_2\psi_2$, the model cannot be represented by the Lagrangian.

We treat the three-tuple (c_0, c_1, c_2) as a point of two-dimensional real projective space RP^2, because two sets which differ from each other by a nonzero factor define the same Gibbs state.

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